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## PHYSICAL MODELING OF ICE COVER DEFORMATION UNDER THE ACTION OF A MOVING LOAD AT LOW SPEED

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### Abstract

The current paper considers the possibility of physical modeling of ice cover deformation under the action of a moving load at low speed. Using an equation for elastic plate oscillations on the foundation of a hydraulic type, it is shown that similarity of a stress and strain state (SSS) of model ice can be achieved but within the scope of the approaches based on a classical theory of Nogid-Shimansky modeling of ice cover. Taking into account certain complications connected with practical implementation of the above method, the applicability of a reduced-thickness ice model developed at NNSTU is investigated. This model uses the ice thickness that intentionally does not comply with the similarity requirements, all other requirements being satisfied, thus providing incomplete similarity of the model. Some disagreements with a Nogid-Shimansky model connected with that are revealed and their influence on the end result is evaluated. The applicability of a thin ice model is investigated in natural cooled model tanks of classical shape used for modeling of load movement at low speed. The results of the experimental investigation of ice cover deformation under the action of a moving load using a model for the natural thin ice are given. The modification of a wave shape and maximum ice deflection depending on change of a movement speed and loading are investigated. The connection of decrease of a ratio of the deflection basin profile area to the outward ice bending profile in front of a moving load with increase of speed at the start of the movement is demonstrated, which can be an evidence of a sharp growth of energy expenses for ice cover deformation when the interaction between the technical facility and ice cannot be considered as quasistatic. An exact evaluation of such expenditures is crucial when designing the ice-breaking facilities clearing a path through floe ice.

**Keywords:** ice, physical modeling, ice model, ice model tank

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## ФИЗИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДЕФОРМАЦИИ ЛЕДЯНОГО ПОКРОВА НАГРУЗКОЙ, ДВИЖУЩЕЙСЯ С МАЛОЙ СКОРОСТЬЮ

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### Аннотация

Рассматривается возможность физического моделирования деформации ледяного покрова от нагрузки, движущейся с малой скоростью. Используя уравнение колебаний упругой пластины на основании гидравлического типа, показано, что подобие напряженно-деформированного состояния в модельном льду может быть достигнуто только в рамках подходов классической теории моделирования ледяного покрова Ногида-Шиманского. Принимая во внимание известные сложности, связанные с практической реализацией этого способа, исследуется применимость модели льда уменьшенной толщины, разработанной в НГТУ.

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Она заключается в преднамеренном несоблюдении подобия льда по толщине при удовлетворении прочих требований, реализуя частичное подобие модели. Показываются возникающие при этом расхождения с теорией Ногида-Шиманского, оценивается их влияние на конечный результат. Исследуется применимость модели тонкого льда в бассейнах с естественным охлаждением классической формы для целей моделирования движения нагрузки с малой скоростью. Приведены результаты экспериментального исследования деформации ледяного покрова под движущейся нагрузкой с использованием модельного тонкого естественного льда. Исследуется изменение формы волны и максимального прогиба льда в зависимости от изменения скорости движения и величины нагрузки. Показана связь уменьшения отношения площади профиля чаши прогибов к профилю выгиба льда перед движущейся нагрузкой с увеличением скорости в начале движения, что может говорить о резком росте энергетических затрат на деформирование ледяного покрова, когда взаимодействие технического средства со льдом уже нельзя рассматривать как квазистатическое. Точное определение этих затрат является критически важным при проектировании ледокольных средств, прокладывающих канал в поле сплошного льда.

**Ключевые слова:** лёд, физическое моделирование, модель льда, ледовый бассейн

## 1. Introduction

Physical modeling has been the most efficient method of investigation of interaction between various marine engineering objects and ice cover in the engineering process. A growing arctic navigation demands building of new, more powerful ice-breakers that can provide not only the icebreaker assistance for heavy-tonnage vessels through the maximum-thickness ice cover but also ensure the movement at a certain commercially reasonable speed. The optimization of the parameters of an icebreaking vessel that would steadily maintain a required speed moving steadily through ice cover with no need of ramming, results in a new advanced design of a vessel shape. [1, 2]. One of the most efficient methods of ice cover breaking is using icebreakers providing an even pressure loading on the ice surface. [3]. An investigation of ice interaction with the icebreaking facilities making a channel through ice at a desired speed requires reconsidering the approaches used in physical modeling, as well as taking into account the dynamic processes occurring in the ice-cover.

A great number of researchers have been building mathematical and physical models of interaction between a moving load and ice cover [4–9]. However, all of those works cover only the cases of near-critical, critical and supercritical speeds of load movement and consider the problems of resonant method of ice breaking by air-cushion vessels, as well as the problems of providing ice bridging and ice airfields. Not too much attention is given to the problems of physical modeling of the ice cover deformation and destruction under the load moving at the speeds specific for the ice-breakers not using a resonant method (hereinafter such speeds are defined as low speeds). In the first place it is due to the fact that till recently the designing of ice-breakers was entirely targeted at achieving the maximum icebreaking capability while breaking ice at a desired speed was not considered. In this connection it becomes relevant to solve the problem of developing some approaches of physical modeling of ice cover deformation and destruction under the action of a load moving at a low speed, and setting the conditions of modeling and similarity criteria, as well as developing techniques of carrying out an experiment in an up-to-date ice model tank.

A work [10] is devoted to the issue of substantiation of the definition “low speed”, and though an investigation of boundary states (when the plastic deformation of ice takes effect) still remains important, the speed can be considered as “low” if bending gravity waves in ice cover do not propagate.

As it is known, the behavior of ice cover under the action of a moving load is well described by an equation for viscoelastic oscillations [4]. The model of an elastic plate gives persistent oscillations at high speeds, which does not agree with the known experimental data. Anyway, an extensive investigation of various mathematical models describing viscoelastic properties of a deformable plate [11, 12] with a use of numerical methods allowed us to arrive at the conclusion that an elastic model at low speeds of load movement yields solutions close to Kelvin-Voigt model; moreover, the lower the speed, the less the disagreement. The solutions obtained were compared with the experimental field data [6], which revealed a good agreement between them. Thus, the conclusion can be made about a small influence of plastic properties of ice at low speeds of load movement; hence, there is a possibility to use a well-developed theory of elasticity for modeling and also the methods of physical modeling of ice as elastic medium.

In this paper, a possibility to carry out modeling of stress and strain state (SSS) of ice cover under the action of a moving load with a use of an equation for oscillations of an infinite elastic plate on a hydraulic type foundation, the Nogid-Shimansky classical theory of ice cover modeling [16] and the modeling using thin natural ice, is investigated. The connection between them is shown, as well as the assumptions that give the possibility to carry out a model experiment with just a partial similarity of the phenomena observed. The results of the experiments in a natural cooled ice model tank of NNSTU are given; they are targeted at investigation of the ice cover behavior under the action of a moving load at low speed, as well as at studying of the applicability of the modeling conditions obtained.

## 2. Modeling conditions

We consider an ice cover as an ideal elastic body. As a mathematical model we use a thin rigid elastic plate on Winkler foundation [13]. It is possible, considering that the depth of ice cover deflection basin and, consequently, the amplitude of a wave propagating in ice cover at low speeds of movement is significantly less than its length. Thus, the plate particles under the action of transverse loading will be subject to just small vertical oscillations, while lateral forces, friction and compressive strength present in ice cover will not be taken into account. In this case the water flow in the vicinity of a plate surface can be assumed as irrotational [14], while the fluid, as a whole — as ideal.

Considering a dynamic problem of ice cover oscillations under the action of load, D.E. Kheysin derives an equation for the elastic plate equilibrium at every instant under the action of external and inner forces [4]:

$$D\nabla^2\nabla^2w + \rho_i h \frac{\partial^2 w}{\partial t^2} = q(x, y, t) + p(x, y, t), \quad (1)$$

where  $\rho_i$  — density of ice;  $D = \frac{Eh^3}{12(1-\mu^2)}$  — cylindrical stiffness of an ice plate;  $E$  — deformation modulus;  $h$  — ice thickness;  $\mu$  — Poisson coefficient;  $q(x, y)$  — external static loading on the plate;  $w(x, y)$  — plate deflection;  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  — Laplace operator;  $p(x, y, t)$  — hydrodynamic loading on the plate.

The above forces acting on a lower edge of the plate for the waves of small amplitude can be defined by a formula:

$$p = -kw - \rho_w \left. \frac{\partial \Phi}{\partial t} \right|_{z=w}, \quad (2)$$

where  $\Phi$  is a potential of fluid rate;  $k = \rho_w g$  — coefficient of Winkler foundation;  $\rho_w$  — water density;  $g$  — an acceleration rate.

Unlike Kheysin, we are going to consider fluid pressure on a lower plate surface not at  $z = 0$ , but at  $z = w$ . In the process of load movement over a plate the latter is subject to deformation as a result of which the water surface is also being deformed. Equating the pressure on a lower surface of the plate with the pressure at the level of the fluid at rest corrects to the condition of permeability of the plate. It may be convenient when looking for analytical solution as it excludes nonlinearity in a differential equation and is acceptable when considering plate deflections and consequently, the movement of fluid particles, as small values. However, when looking for criteria of similarity in physical modeling there is no need in such assumption.

In the equation under consideration only vertical forces are acting on the plate, while pressure travel causes but its vertical deflections. In this case it can be assumed that the movement of fluid in the vicinity of a lower surface of the plate would be also vertical only.

$$\vec{v}_x, \vec{v}_y = 0 \Big|_{z=w}, \quad (3)$$

where  $\vec{v}_x$  и  $\vec{v}_y$  are projections of a flow rate vector on axes  $x$  and  $y$ .

If the stress and strain state (SSS) of an ice cover model can be described with the same equilibrium equation (1) used for natural ice, then the terms of the equation for the model must be related to the corresponding terms of an equation for natural ice through appropriate modeling scales:

$$\begin{aligned} D_n \nabla^2 \nabla^2 w_n &= \frac{\lambda_D \lambda_w}{\lambda_l^4} D_m \nabla^2 \nabla^2 w_m; \quad (\rho_i)_n h_n \frac{\partial^2 w_n}{\partial t_n^2} = \frac{\lambda_{\rho_i} \lambda_w \lambda_h}{\lambda_t^2} (\rho_i)_m h_m \frac{\partial^2 w_m}{\partial t_m^2}; \\ (\rho_w)_n g w_n &= \lambda_{\rho_w} \lambda_w (\rho_w)_m g w_m; \quad (\rho_w)_n \left. \frac{\partial \Phi_n}{\partial t_n} \right|_{z_n=w_n} = \frac{\lambda_{\rho_w} \lambda_v \lambda_w}{\lambda_t} (\rho_w)_m \left. \frac{\partial \Phi_m}{\partial t_m} \right|_{z_m=w_m}; \\ q_n(x_n, y_n, t_n) &= \frac{\lambda_P}{\lambda_l^2} q_m(x_m, y_m, t_m). \end{aligned} \quad (4)$$

where  $\lambda_l, \lambda_w, \lambda_h, \lambda_D, \lambda_{\rho_i}, \lambda_{\rho_w}, \lambda_t, \lambda_v, \lambda_P$  are model scales of linear dimensions, plate deflections, ice thickness, cylindrical stiffness of plate, ice density, water density, time, speed and forces, respectively; henceforward, the indices  $n$  and  $m$  are used to distinguish an element as describing either nature or a model, respectively.

Then, an equation of equilibrium for a natural ice plate can be written as follows:

$$\begin{aligned} \frac{\lambda_D \lambda_w}{\lambda_l^4} D_m \nabla^2 \nabla^2 w_m + \frac{\lambda_{\rho_i} \lambda_w \lambda_h}{\lambda_t^2} (\rho_i)_m h_m \frac{\partial^2 w_m}{\partial t_m^2} + \lambda_{\rho_w} \lambda_w (\rho_w)_m g w_m + \frac{\lambda_{\rho_w} \lambda_v \lambda_w}{\lambda_t} (\rho_w)_m \left. \frac{\partial \Phi_m}{\partial t_m} \right|_{z_m=w_m} &= \\ = \frac{\lambda_P}{\lambda_l^2} q_m(x_m, y_m, t_m). \end{aligned} \quad (5)$$

We divide both parts of the equation by  $\lambda_w$ :

$$\begin{aligned} \frac{\lambda_D}{\lambda_l^4} D_m \nabla^2 \nabla^2 w_m + \frac{\lambda_{\rho_i} \lambda_h}{\lambda_t^2} (\rho_i)_m h_m \frac{\partial^2 w_m}{\partial t_m^2} + \lambda_{\rho_w} (\rho_w)_m g w_m + \frac{\lambda_{\rho_w} \lambda_v}{\lambda_t} (\rho_w)_m \left. \frac{\partial \Phi_m}{\partial t_m} \right|_{z_m=w_m} &= \\ = \frac{\lambda_P}{\lambda_l^2 \lambda_w} q_m(x_m, y_m, t_m). \end{aligned} \quad (6)$$

Let's consider the coefficients preceding the terms of an equation derived. When carrying out model tests, the difference of natural and model water densities is neglected, which allows us to fulfil modeling of a third term of the equation describing an effect of Winkler foundation:

$$\lambda_{\rho_w} = 1. \quad (7)$$

A model scale of cylindrical stiffness can be written in a following way:

$$\lambda_D = \lambda_E \lambda_h^3 \frac{1 - \mu_n^2}{1 - \mu_m^2}. \quad (8)$$

To derive an equation for deflection of model ice plate equivalent to that for natural ice plate deflection, the coefficients preceding the terms of an equation (6) that describes an interaction of the model scales should not affect the equation, i.e.:

$$\frac{\lambda_E \lambda_h^3}{\lambda_l^4} \cdot \frac{1 - \mu_n^2}{1 - \mu_m^2} = 1, \quad \frac{\lambda_{\rho_i} \lambda_h}{\lambda_t^2} = 1, \quad \frac{\lambda_{\rho_w} \lambda_v}{\lambda_t} = 1, \quad \frac{\lambda_P}{\lambda_l^2 \lambda_w} = 1. \quad (9)$$

If a model satisfies the conditions of ice plate deflection similarity:

$$\lambda_w = \lambda_l = \lambda, \quad (10)$$

then a coefficient preceding the right-hand-side term of an equation (6) that describes an action of the external load applied to an ice plate, is rewritten in a following way:

$$\lambda_P = \lambda_l^2 \lambda_w = \lambda^3. \quad (11)$$

The above expression is a dynamic model scale known from the classical theory of similarity [15] and is a condition of modeling satisfying the Newton laws.

A coefficient preceding a second term of an equation (6) is also a condition of similarity of inertial forces of ice if the density of model ice material is the same as that of natural ice cover:

$$\lambda_{\rho_i} = 1, \quad (12)$$

while ice thickness conforms to the geometrical model scale:

$$\lambda_h = \lambda, \quad (13)$$

we obtain a condition presenting a time model scale:

$$\lambda_t = \sqrt{\lambda_h \lambda_{\rho_i}} = \sqrt{\lambda}. \quad (14)$$

Considering a coefficient of a fourth term of an equation (6) that describes the kinematic fluid pressure on a lower surface of the plate, and taking into account (7) and (13), we get an expression for kinematic model scale [15]:

$$\lambda_v = \frac{\lambda_t}{\lambda_{\rho_w}} = \sqrt{\lambda}. \quad (15)$$

When using as a model ice cover the material with a Poisson coefficient same as for natural ice  $\lambda_\mu = 1$ , we get a condition of similarity of elastic forces of a plate, described by a first term of the equation (6):

$$\frac{\lambda_E \lambda_h^3}{\lambda_t^4} = 1, \text{ or in a different form:}$$

$$\lambda_h = \lambda^{4/3} \lambda_E^{-1/3}. \quad (16)$$

Taking into account (13), we get:

$$\lambda_E = \lambda. \quad (17)$$

All of the conditions considered above make up a constituent part of the known criteria of ice modeling developed by Nogid and Shimansky [16] (the so-called classical theory of ice cover modeling):

$$\lambda_{\rho_w} = \lambda_{\rho_i} = \lambda_\mu = \lambda_f = 1, \lambda_h = \lambda, \lambda_E = \lambda_\sigma = \lambda, \lambda_v = \sqrt{\lambda}, \lambda_P = \lambda^3, \quad (18)$$

where  $\lambda_f$  and  $\lambda_\sigma$  are model scales of a coefficient of friction and critical flexural stresses.

Thus, from the results of the performed analysis it follows that when carrying out the model experiments on ice breaking under the action of a load moving at low speed, to achieve similarity of a stress and strain state (SSS) of ice cover described by an equation (1) is possible if the conditions of classical theory of ice cover modeling are satisfied.

However, the actual fulfilment of those conditions meets with significant difficulties connected with both an insufficient knowledge of the processes of natural ice cover breaking and development of a suitable material for model ice. A more detailed description of that can be found in [17, 18]. The most promising is a composite ice cover model made of granulated polyethylene of high pressure ('GP-ice') [19, 20]. However, the properties of such model subjected to static breaking have not been studied well enough to speak of its application for the dynamic problems.

An ingenious method of physical modeling of vessel movement through ice based on a similar approach to determination of similarity criteria is suggested in [21]. A static equation for equilibrium of an elastic isotropic plate on Winkler foundation is used:

$$D\nabla^2\nabla^2 w = q(x, y) - kw. \quad (19)$$

An equation (19) is considered as a mathematical model giving an adequate description of SSS of ice cover in the process of ice cover breaking by an icebreaking vessel moving through heavy ice, which determines its sufficiency for the purpose of identifying similarity criteria in physical modeling of ice cover destruction by an icebreaker. In this case the rate of sailing is low enough, and the interaction of the vessel with ice can be assumed as quasistatic [22]. On the basis of a generalized mathematical description of natural and model ice cover deformation with an equation (19), the following similarity criteria are found:

$$\lambda_{\rho_w} = \lambda_{\rho_i} = \lambda_\mu = \lambda_f = 1, \lambda_h = \lambda^{4/3}, \lambda_v = \sqrt{\lambda}, \lambda_P = \lambda^3. \quad (20)$$

As a model material the natural ice with no additives is used. Such method assumes intentional non-compliance with the geometrical scale when modeling ice thickness, sacrificing similarity of mass properties of the fragments for modeling of inner elastic forces and ultimate strength limit. Such approach became possible due to experimental division of total ice resistance of a vessel into a number of components:

$$R = R_f + R_r + R_w, \quad (21)$$

where  $R_f$  is resistance due to breaking of continuous field ice;  $R_r$  is the ice fragment resistance to the vessel movement;  $R_w$  is water resistance.

In this case at first the tests are carried out in natural unbroken ice and then — using the ice fragments obtained. As a result, the ice breaking resistance is obtained  $R_f$ . After that the tests are fulfilled with a use of high-pressure polyethylene plates imitating ice fragments, and then the tests are carried out in clear, ice-free water. As a result, the total ice resistance (21) as a sum of its components was obtained. More detailed information about the above method can be found in [18, 22].

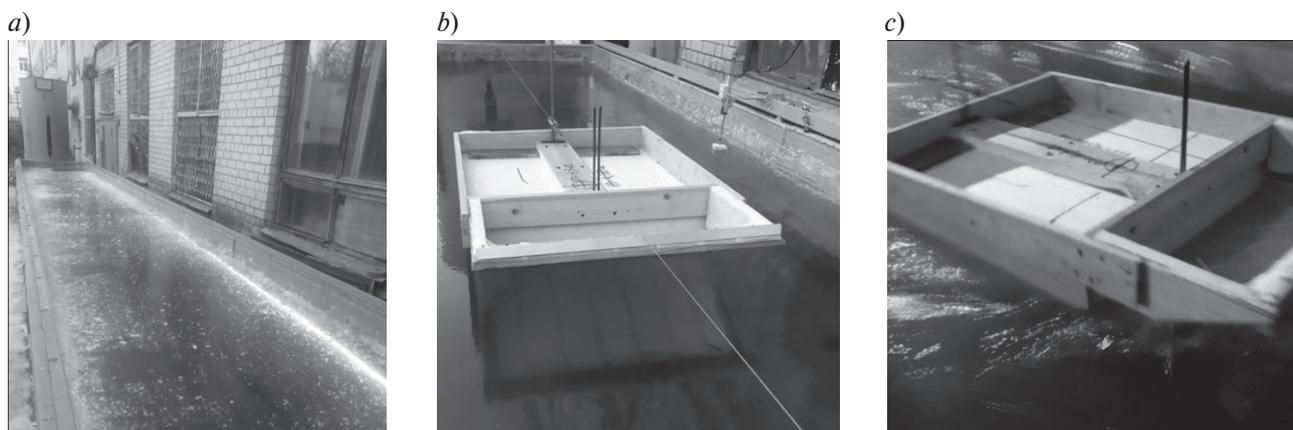
The actual experience of applying the model under consideration for solving various problems of ice resistance of vessels allowed us to speak about efficiency of the above method not only when using it at maximum ice thickness but also at other real speeds of movement of displacement ships through ice. [23]. Thus, the method can be considered as promising when modeling ice cover breaking under the action of a load moving at low speed.

It should be noted that the use of the conditions (20) while modeling the ice cover broken under the action of a moving load according to an equation (1) leads to failure to fulfil a condition (14) expressing similarity of ice cover inertia forces. However, successful application of the method for testing of the ice-breaker models at different speeds of movement through ice can indicate that the power costs connected with inertia of an ice plate are insignificant as compared with the total expenses. Hence, it follows that in real conditions when modeling ice cover deformation with an equation (6) a second term describing inertial properties of ice can be neglected because it is smaller than other terms.

Thus, physical modeling of ice cover breaking under the action of a load moving at low speed, describing SSS with an equation of equilibrium of an elastic isotropic thin plate on Winkler foundation (1) is possible if a condition (20) is fulfilled with a use of thin natural ice as the model material. This method requires little workload and allows carrying out the experiments at small model scales in natural cooled tanks, which significantly reduces the costs of experiments.

### 3. Experimental investigations

To study the dynamic processes developing in ice cover under the action of an even load at different speeds of movement, and the applicability range of the assumptions considered in the previous section, a number of experiments were implemented. The experiments were carried out in an ice tank of NNSTU (Fig. 1, *a*) at negative air temperature; the ice was built up under the action of natural cold. The length of the tank was equal to 16 m, the depth made up 1.6 m, while the depth of the tank was 0.8 m. As a load a platform made of plastic foam



**Fig. 1.** Experiments on investigating the action of a moving load on the model ice: *a* — ice tank of NNSTU; *b* — load movement at low speed; *c* — ice breaking at a speed close to resonant

of a rectangular shape (Fig. 1, *b*) was used, its plan dimensions being equal to  $0.6 \times 0.5$  m. A lower surface of the platform had a fluoroplastic coating to reduce friction. The platform itself weighed 6.32 kg and was loaded with ballast up to 21 kg.

The plastic foam had a required density and hardness that on the one hand provided complete fitness of the experimental device to the deformable ice and distribution of loading, and on the other hand, provided sufficient strength and integrity of the device when carrying out the experiments. The platform was secured in a gravity towing system with a real-time speed recorder. Towing of the model was fulfilled with a use of a towing load. The speed of movement at the start of the experiments turned out to be uneven due to varying roughness of ice surface. Wetting of the ice surface with water helped to balance the speed.

Since resonance was achieved at exceedingly high speeds due to the natural depth of the tank, a well-known effect of the resonant speed decrease with decrease of the tank depth was used [4]. In shallow water the velocity of bending-gravity wave propagation at which a phenomenon of the parametric resonance is achieved, does not depend on ice characteristics and is determined in the following way:

$$v_r = \sqrt{gH}, \quad (22)$$

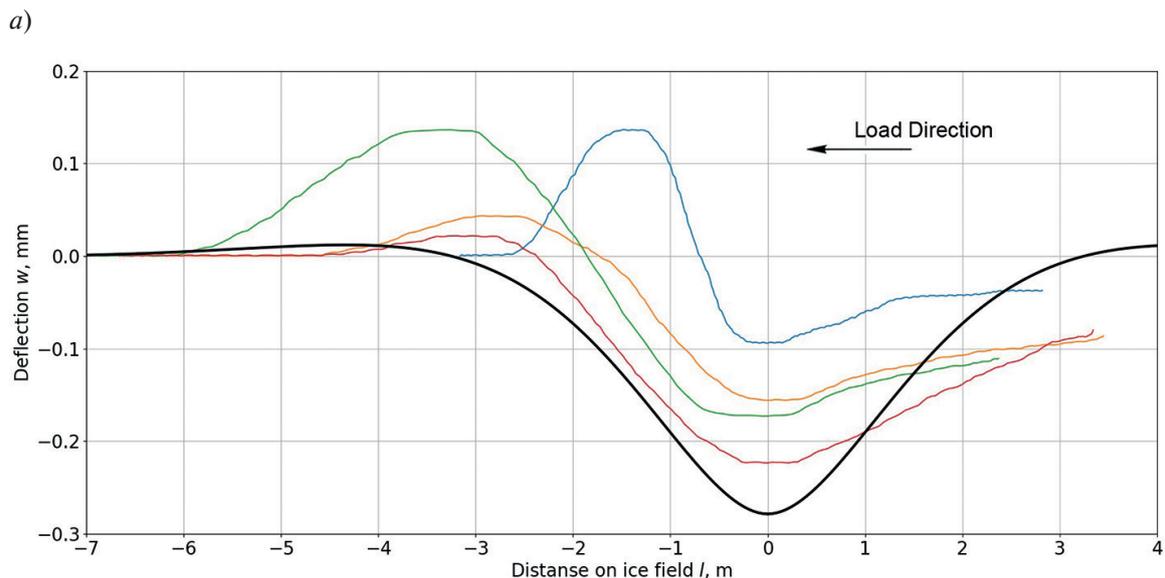
where  $H$  is depth of a tank.

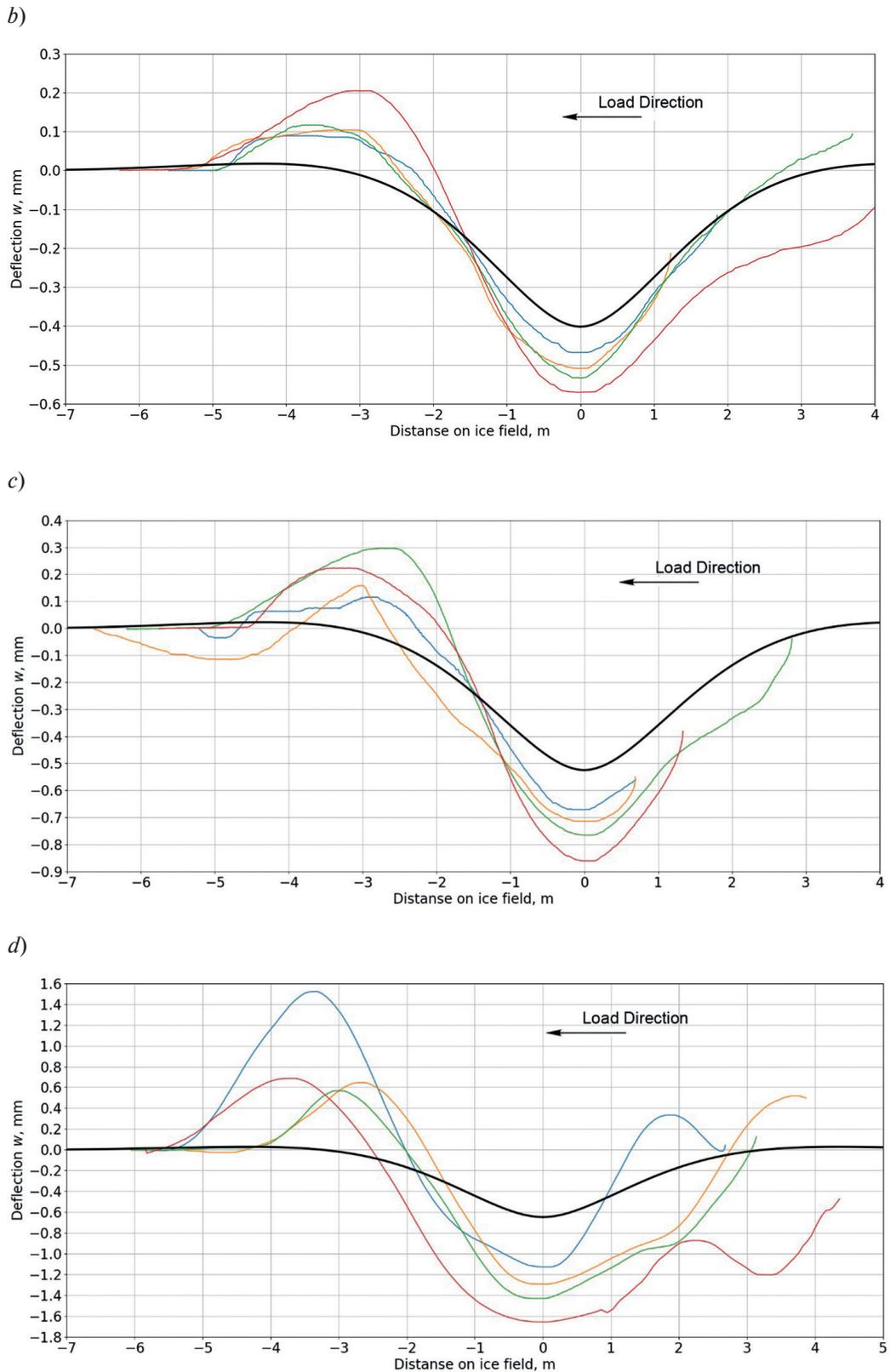
To obtain such an effect, a technique of freezing on “a second bottom” was used. The tank water was drained by 0.3 m below the standard level, and the ice was built up until the thickness of 50 mm was reached. After that water was poured over the ice up to the standard level. Thus, a shallow water reservoir ( $H = 0.3$  m) in which a resonant speed did not exceed 1.7 m/sec., was reproduced. Subsequently, an ice layer of thickness 25 mm was built up over the standard level to be used for load movement.

Following the end of the process of building up ice, the sheet of ice was cut off along the tank sides (excluding the ends) so that to achieve free vertical movement of the ice edges relative to the reservoir walls. Thus, a case of cylindrical ice plate bending under the action of a moving load, i.e. propagation of a plane bending gravity wave in shallow water, was reproduced. After preparation of an ice sheet and mounting the transducers in place, a model of predetermined weight was placed on ice and was towed over the surface at a certain speed. (Fig. 1, *b*). Near the tank ends the cuts did not reach the walls by 1.5 m, thus leaving some area to be used as an acceleration path that along with the test loads allowed us to obtain steadiness of the process in the testing area.

In the course of the experiments a number of the following factors were varied: movement speed; weight of a moving load; ice thickness. The following parameters were registered: the speed of load movement obtained by using a routine system of towing speed measurement (a photo pulse type transducer, an analog-to-digital converter); ice bending recorded in time using a linear movement potentiometer transducer installed in the middle of the tank.

Each experiment recorded shape of a wave moving along with the load past the transducer. At constant speed of load movement there was an obvious connection between the time of the process duration and a linear coordinate of the current bending location relative to the movement transducer. The profiles obtained at different loading are given in Fig. 2.





**Fig. 2.** Profiles of waves in ice cover during the movement of a load weighing: *a* – 11.32 kg: — static profile; — speed 0.9 m/sec.; — speed 1.38 m/sec.; — speed 0.72 m/sec.; — speed 0.15 m/sec.; *b* – 16.32 kg: — static profile; — speed 1.00 m/sec.; — speed 0.53 m/sec.; — speed 0.30 m/sec.; — speed 0.25 m/sec.; *c* – 21.32 kg: — static profile; — speed 0.5 m/sec.; — speed 0.88 m/sec.; — speed 0.42 m/sec.; — speed 0.22 m/sec.; *d* – 27.32 kg: — static profile; — speed 1.65 m/sec.; — speed 0.8 m/sec.; — speed 0.71 m/sec.; — speed 0.45 m/sec.

#### 4. Results of the experiments and discussion

To obtain qualitative characteristics of an ice plate deformation process under the action of a moving load, the results obtained were compared to the static ones theoretically calculated for the conditions similar to experimental. The ice sheet was assumed as a cylindrically bent plate of infinite length on elastic Winkler foundation. Such problem was reduced to solving an equation for beam-strip on Winkler foundation subjected to concentrated loading. An expression for deflections of a right half of beam [24] was converted for a beam-strip case in accordance with the recommendations [25] and written for both halves:

$$w = -\frac{P}{b_i} \frac{\alpha}{2k} e^{-\alpha|x|} [\cos(\alpha|x|) + \sin(\alpha|x|)], \quad (23)$$

where  $x$  is a coordinate of beam;  $w$  are deflections of the beam-strip;  $P \frac{1}{b_i}$  — loading applied to beam-strip;

$P$  — gravity due to loading;  $b_i = 1.5$  m — width of ice sheet;  $k = \rho_{wg} = 9810$  N/m<sup>3</sup> — coefficient of elastic foundation;  $\alpha = \sqrt[4]{\frac{k}{4D}} = 0.726$  m<sup>-1</sup> — a parameter of beam and elastic foundation;  $D = \frac{Eh^3}{12(1-\mu^2)} = 8834$  N·m is

the cylindrical stiffness of plate (obtained for ice thickness  $h_i = 0.025$  m);  $E = 6 \times 10^9$  Pa is an elastic modulus of ice [18];  $\mu = 0.34$  is a Poisson coefficient for ice [18]. The calculations results are shown in Fig. 3; these curves are also indicated by solid lines in the graphs showing the profiles of ice bending waves (Fig. 2).

The results of the experiment analysis (Fig. 2) allowed us to reveal interconnections between the wave length, a shape of basin deflections, the load weight, a speed at constant ice thickness and the tank depth (Fig. 4). The same values were obtained as a result of analysis of the calculated static deflections (Fig. 3), and superimposed on the plotted experimental data (Fig. 4) at zero speed of load movement. In those cases when the experiment failed to make a complete record of the bowl of deflections, the obtained values were extrapolated to the point of intersection with the level of the calm state (mark 0 on the y-axis of the graphs in Fig. 2). The analysis of the dependences obtained showed that at low speeds of movement an increase of ice bending becomes noticeable as compared with the statics in most of the cases, except when the weight of load is 11.32 kg; the more the loading the more significant that difference (Fig. 5). The change of a trough area at the start of the movement as compared to a static state is insignificant, as a rule. The ratio of trough and bulge areas before loading decreases sharply when changing from a static state to dynamic; however, when the speed reaches 20–30 % of resonant speed, no significant changes are observed. A qualitative difference of

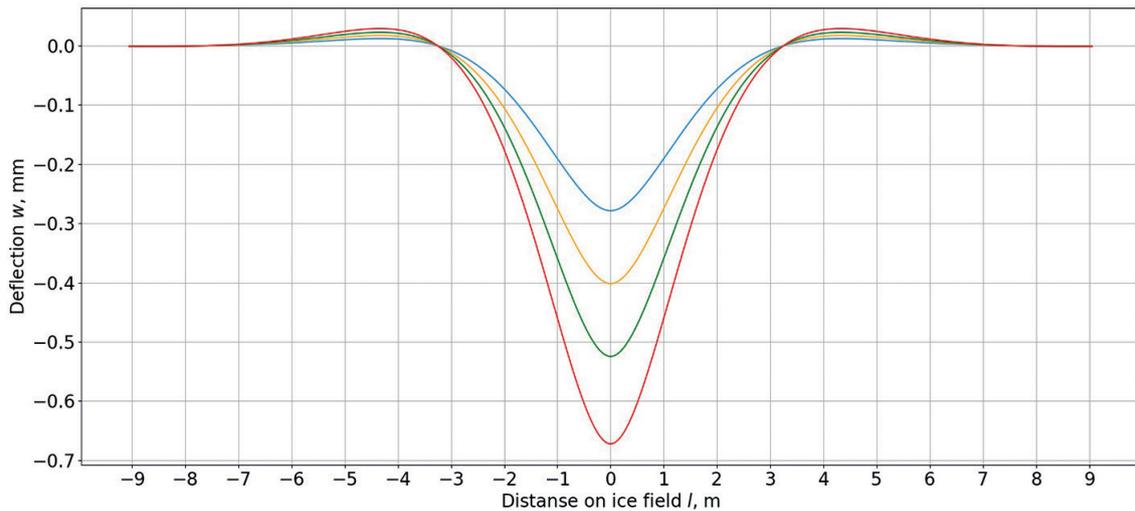


Fig. 3. The results of calculation of static cylindrical ice plate bending:  
 — 27.32 kg; — 21.32 kg; — 16.32 kg; — 11.32 kg

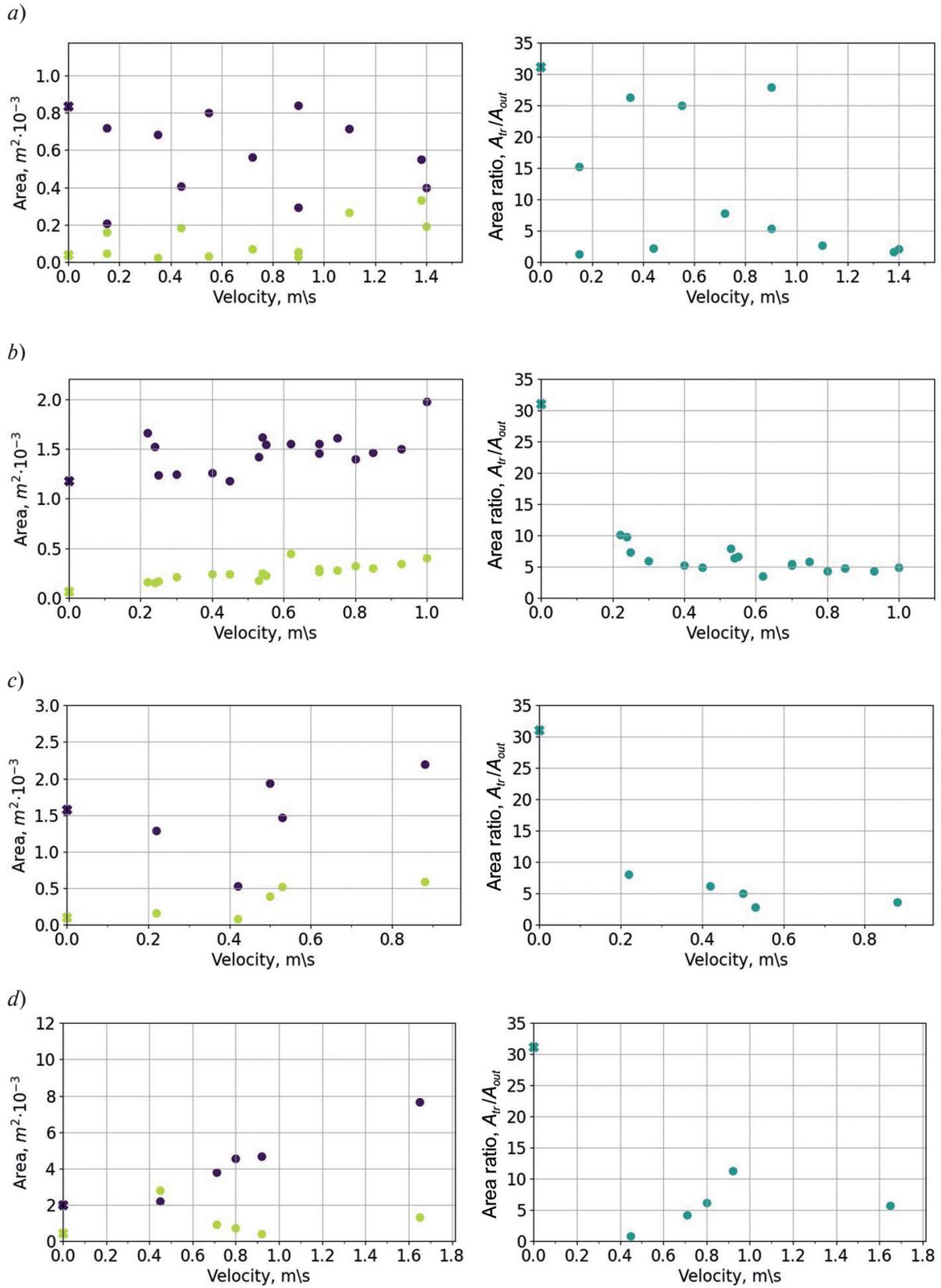


Fig. 4. Characteristics of a wave profile shape for a case of a moving load weighing:

a — 11.32 kg; b — 16.32 kg; c — 21.32 kg;

d — 27.32 kg; legend: ● area of outward bending  $A_{out}$ ; ● trough area  $A_{tr}$ ; ● area ratios  $A_{tr}/A_{out}$

a flexure basin shape depending on the speed of load movement was found. At a relatively low speed (about 20–30 % of resonant speed) a flexure basin shape was not subject to significant distortion, while as compared to the static state in the front part of the flexure basin (from the side towards which the load was moving), a strong outward ice bending occurred from the zero level upwards. The height of that outward bending made up 20–30 % of maximum deflection in the area of loading application, while the length of outward bending was within 20–30 % of the flexure basin length.

A steady tendency for increase of the outward bending area with increase of load movement speed for the load weights of 11.32 kg, 16.32 kg and 21.32 kg was observed; for the load weight of 27.32 kg the area of outward bending was decreasing while upon passing the minimum began growing too.

A ratio of the area of flexure basin to that of the outward bending before loading can be considered as a measure of power expenses for deformation of ice cover: the less the ratio the more the expenses. For each variant of loading weight the graphs were plotted showing the ratio variation depending on the speed of movement (Fig. 4). A tendency for its decrease with speed growth was found; moreover, the most significant decrease in the indicator occurs at low speeds of the load (up to 20–30% of the resonant speed), starting from the calculated static case. It is obvious that a change of outward ice bending area before loading contributes most of all to variation of that ratio.

A shape of flexure basin in the back and that in the front part were not symmetric; the symmetry was observed just in few experiments. A correlation between the change of ice plate bending shape and the maximum deflection with increase of a load movement speed (that matches the known experimental field data [8]) is evident. Such effect is most noticeable at low speeds of load movement up to 20–30 % of resonant speed.

Registration of deflections in time as well as the speed of load movement allowed us to approximately estimate the length of a flexure basin moving together with a load, which turned out to be approximately equal to the length of wave of cylindrical ice bending, i.e. — 6–7 m. As the deflection transducer was recording deflections approximately in the middle of the model tank, at a distance of 9 m from the back wall of the tank, it was obvious that a reflected wave was superimposed on the profile of an initial wave. Therefore, at high speed of load movement a flexure basin was not quite symmetrical. It was especially noticeable at the speeds of movement from 0.7 of resonant speed to the values of the resonant speed itself. A peculiar feature of change of a flexure basin shape displays itself in appearance within the basin flexure profile similar to that obtained at low speeds of a number of short waves — up to four waves of length 0.2 from the main flexure basin, and of height 0.2 from the maximum flexure. The tests carried out at a speed close to resonant proved a phenomenon observed in nature, i.e. a sharp wave elevation above the zero level behind the flexure basin of a

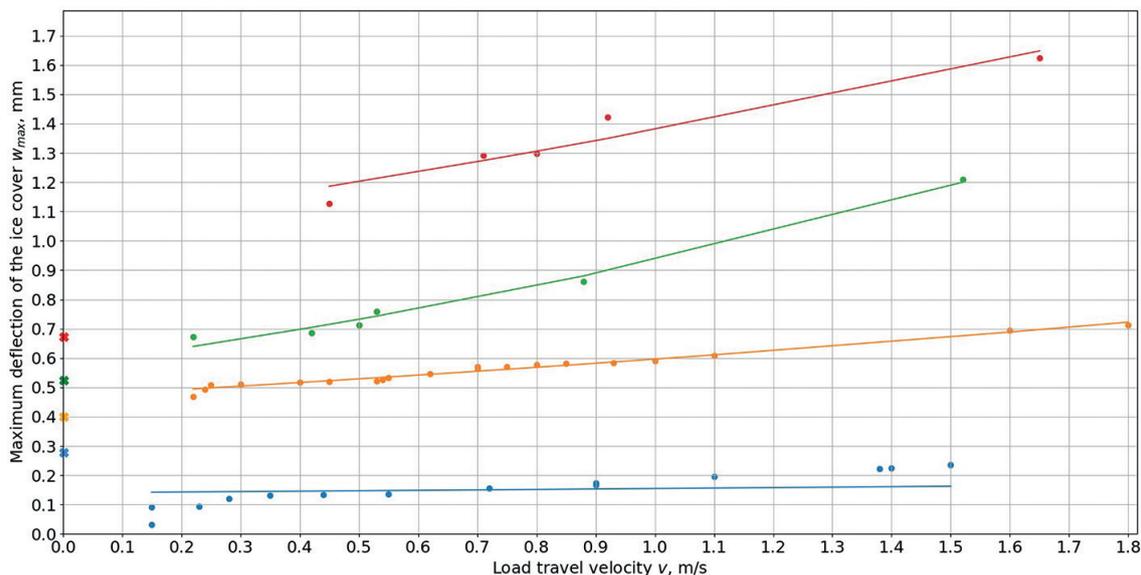


Fig. 5. Dependence of maximum ice deflection on weight and speed of load movement: ♦♦ 27.32 kg,  $w_{\max} = 1.059e^{0.264v}$ ; ★★ 21.32 kg,  $w_{\max} = 0.589e^{0.452v}$ ; ○○ 16.32 kg,  $w_{\max} = 0.521e^{0.100v}$ ; ■■ 11.32 kg,  $w_{\max} = 0.140e^{0.100v}$

moving load. The profile of this wave had a different shape in each of the experiments, the repeatability being absent at all. Obviously, such effect occurred quite at random due to reflection of a bending gravity wave from the end wall of the reservoir.

The analysis of the experimentally obtained relationships between deflection values and the values of loading and speed revealed poor dependence of a maximum flexure on speed of movement at speeds exceeding 20–30 % of resonant speed. It is particularly insignificant at small weight of a moving load. (Fig. 5). Within a reasonable statistical error, the dependence on speed up to resonant speed is absent at loading weight of 11 and 16 kg. At weight of 21 kg the dependence becomes noticeable and deflection grows 1.5 times at resonant speed, as compared with the deflection occurring at speed equal to 0.1 of resonant speed. The greatest effect is observed when a load weighing 27 kg is moved. Deflection grows twice as much at resonant speed comparing to that at low speed. It is in agreement with V.M. Kozin's data also demonstrating the significant dependence of resonant deflection on weight of a moving load [7].

## 5. Conclusions

The possibility of modeling of dynamic phenomena occurring in ice cover under the action of load movement in natural cooled model ice tanks of classical shape is demonstrated. An effect of ice surface bending in front of a moving flexure basin was achieved, which can be considered as one of the validity factors when building theoretical models and their resolutions. An increase of ice cover flexure basin with increase of a load movement speed (at low speed up to 30 % of resonant speed) in comparison with a case of the calculated static loading was revealed. The connection between decrease of ratio of flexure basin profile area to the profile of ice bending in front of a moving load and increase of speed is shown. This fact points out at the theoretical probability of sharp growth of power expenses for ice breaking at increase of movement speed of an icebreaking facility when interaction with ice cannot be considered as quasistatic. A more detailed study of that effect requires additional experimental investigations at load movement speed up to 30 % of resonant speed. The given experiment scheme can serve as an example of a test for verification of conformity of a physical model of ice cover to the problems formulated within the scope of dynamics theory.

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## References

1. Gramuzov E.M., Tikhonova N.E. Justification of design characteristics of river icebreakers. *Transactions of the NNSTU n.a. R.E. Alekseev*. 2013, 3 (100), 195–199 (In Russian).
2. Tikhonova N.E. Optimizing of the basic elements and shell architecture of an icebreaker depending on the ice conditions. *Transactions of NNSTU n.a. R.E. Alekseev*. 2014, 5 (107), 302–308 (In Russian).
3. Dvoichenko Y.A., Semenova N.M. The influence of the area size and the method of transverse load application to the ice cover surface on the ice failure efficiency. *IOP Conference Series: Materials Science and Engineering*. 2021, 1079, 7, 072020.
4. Hejsin D.E. Ice cover dynamics. *Leningrad, Gidrometeoizdat*, 1967. 215 p. (In Russian).
5. Cherkesov L.V. Surface and Internal Waves. *Kiev, Naukova Dumka*, 1973. 247 p.
6. Squire V.A., Hosking R.J., Kerr A.D., Langhorne P.J. Moving loads on ice plates. *Berlin, Springer Science & Business Media*, 1996, 45. 244 p.
7. Kozin V.M., Zhestkaya V.D., Pogorelova A.V., Chizhiumov S.D., Lzhabrailov M.P., Morozov V.S., Kustov A.N. Applied problems of ice cover dynamics. *Moscow, Akademiya Estestvoznaniya*, 2008. 329 p. (In Russian).
8. Ivanov K.E. Load capacity of the ice cover and the construction of roads on the ice. *Leningrad, Glavsevmorput'*, 1949. 182 p. (In Russian).
9. Nevel D.E. Moving loads on at floating ice sheet. Research report (Cold Regions Research and Engineering Laboratory (U.S)), 1968, 261. 13 p.
10. Zuev V.A., Lu Yang, Dvoichenko Yu.A., Sebin A.S. Approaches to estimating the ice cover crushing during load moving above it with a low speed. *Transportnye Sistemy*. 2020, 3 (17), 41–47 (In Russian).
11. Zhestkaya V.D. Numerical solution of the problem of the load movement on the ice cover. *Prikladnaya Mekhanika i Tekhnicheskaya Fizika*. 1999, 40, 4, 243–248 (In Russian).

12. Kozin V.M., Pogorelova A.V. Effect of the viscosity properties of ICE on the deflection of an ICE sheet subjected to a moving load. *Journal of Applied Mechanics and Technical Physics*. 2009, 50, 3, 484–492.
13. Reddy J.N. Theory and analysis of elastic plates and shells. CRC press, 2006. 547 p.
14. Landau L.D., Lifshitz L.D. Theoretical physics. Volume VI. Hydrodynamics. Moscow, Nauka, 1986. 736 p. (In Russian).
15. Epstein L.A. Methods of the dimensions and similarity theory in problems of ship hydromechanics. Leningrad, Sudostroeniye, 1970. 208 p. (In Russian).
16. Kashatelyan V.I., Poznyak I.I., Ryvlin A.Y. Ice resistance to ship movement. Leningrad, Sudostroeniye, 1968. 238 p. (In Russian).
17. Gramuzov E.M., Dvoichenko Yu.A., Zuev V.A., Sebin A.S. Ice cover modeling in predicting the interaction of ships and offshore structures with ice. *Mechanics of Solids*. 2021, 56, 2, 179–188. doi: 10.3103/S0025654421020084
18. Ionov B.P., Gramuzov E.M. Ice propulsion of ships. 2nd edition. St. Petersburg, Sudostroeniye, 2013. 504 p. (In Russian).
19. Dvoichenko Yu.A., Zuev V.A., Sebin A.S. On application of composite ice model in ice sheet simulation. *Transactions of the Krylov State Research Center*. 2019, S2, 59–62. doi:10.24937/2542–2324–2019–2-S-I-59–62
20. Sebin A.S., Dvoichenko Y.A. The new areas of the ice cover modeling using the composite model ice “GP-ice”. *IOP Conference Series: Materials Science and Engineering*. 2021, 1079, 7, 072017.
21. Zuev V.A., Rabinovich M.E., Gramuzov E.M., Dvoichenko Yu.A. On modeling of the ship movement in level ice field. *Teoriya i Prochnost' Ledokol'nogo Korablya: mezhvuz. sb. Gorky*, 1978, 22–25 (In Russian).
22. Zuev V.A. Means of prolonging navigation on inland waterways. Leningrad, Sudostroeniye, 1986. 207 p. (In Russian).
23. Zuev V.A., Gramuzov E.M., Dvoichenko Y.A., Sebin A.S. The experimental and theoretical investigations of the “GP-ice” Use in Ships Model Tests. *IOP Conference Series: Earth and Environmental Science*. 2020, 459, 4, 042071.
24. Davydov V.V., Mattes N.V., Sivercev I.N. Study guide on the strength of inland navigation vessels. Moscow, Rechnoj Transport, 1958. 755 p. (In Russian).
25. Papkovich P.F. Works on the construction mechanics of the ship. In 4 volumes. Vol. 3. Complex bending of rods and plates. Leningrad, Sudpromgiz, 1962. 527 p. (In Russian).

## Литература

1. Грамузов Е.М., Тихонова Н.Е. Обоснование проектных характеристик речных ледоколов // Труды НГТУ им. ПЕ Алексеева. Н. Новгород, 2013. № 3 (100). С. 195–199.
2. Тихонова Н.Е. Оптимизация основных элементов и формы корпуса ледокола в зависимости от ледовых условий // Труды НГТУ им. ПЕ Алексеева. Н. Новгород, 2014. № 5 (107). С. 302–308.
3. Dvoichenko Y.A., Semenova N.M. The influence of the area size and the method of transverse load application to the ice cover surface on the ice failure efficiency // IOP Conference Series: Materials Science and Engineering. IOP Publishing, 2021. Vol. 1079, N 7. P. 072020.
4. Хейсин Д.Е. Динамика ледяного покрова. Л.: Гидрометеиздат, 1967. 215 с.
5. Черкесов Л.В. Поверхностные и внутренние волны. Киев: Наукова думка, 1973. 247 с.
6. Squire V.A., Hosking R.J., Kerr A.D., Langhorne P.J. Moving Loads on Ice Plates. Springer Science & Business Media, 1996. Vol. 45. 244 p.
7. Козин В.М. и др. Прикладные задачи динамики ледяного покрова. М.: Академия естествознания, 2008. 329 с.
8. Иванов К.Е., Песчанский И.С. Грузоподъемность ледяного покрова и устройство дорог на льду. Ленинград: Главсевморпуть, 1949. 181 с. (Труды Арктического научно-исследовательского института Главного управления Северного Морского пути при Совете министров СССР; Т. 23).
9. Nevel D.E. Moving loads on at floating ice sheet. Research report (Cold Regions Research and Engineering Laboratory (U.S)). 1968. Vol. 261. 13 p.
10. Зув В.А., Лу Ян, Двойченко Ю.А., Себин А.С. Подходы к оценке разрушения ледяного покрова при движении над ним нагрузки с малой скоростью // Транспортные системы. Н. Новгород, 2020. № 3 (17). С. 41–47.
11. Жесткая В.Д. Численное решение задачи о движении нагрузки по ледяному покрову // Прикладная механика и техническая физика. 1999. Т. 40, № 4. С. 243–248.
12. Козин В. М., Погорелова А.В. Влияние вязкостных свойств льда на прогиб ледового покрова при движении по нему нагрузки // Прикладная механика и техническая физика. 2009. Т. 50, № 3. С. 147–157.
13. Reddy J.N. Theory and analysis of elastic plates and shells. CRC press, 2006. 547 p.
14. Ландау Л.Д., Лифшиц Л.Д. Теоретическая физика. Том VI. Гидродинамика. М.: Наука, 1986. 736 с.
15. Эпштейн Л.А. Методы теории размерностей и подобия в задачах гидромеханики судов. Л.: Судостроение, 1970. 208 с.

16. Каштелян В.И., Позняк И.И., Рывлин А.Я. Соппротивление льда движению судна. Л.: Судостроение, 1968. 238 с.
17. Грамузов Е.М., Двойченко Ю.А., Зуев В.А., Себин А.С. Моделирование ледяного покрова при прогнозировании взаимодействия судов и морских сооружений со льдом // Известия Российской академии наук. Механика твёрдого тела. 2021. № 2. С. 51–62. doi:10.31857/S0572329921020094
18. Ионов Б.П., Грамузов Е.М. Ледовая ходкость судов. 2 издание. СПб.: Судостроение, 2013. 504 с.
19. Двойченко Ю.А. Себин А.С., Зуев В.А. К вопросу о моделировании ледяного покрова с использованием композитной модели льда // Труды Крыловского государственного научного центра. 2019. № S2. С. 59–62. doi:10.24937/2542-2324-2019-2-S-1-59-62
20. Sebin A.S., Dvoichenko Y.A. The New Areas of the Ice Cover Modeling Using the Composite Model Ice “GP-ice” // IOP Conference Series: Materials Science and Engineering. 2021. Vol. 1079, N 7. P. 072017.
21. Зуев В.А., Рабинович М.Е., Грамузов Е.М., Двойченко Ю.А. К вопросу о моделировании движения судна в сплошном ледяном поле // Теория и прочность ледокольного корабля: межвуз. сб. Горький, 1978. С. 22–25.
22. Зуев В.А. Средства продления навигации на внутренних водных путях. Л.: Судостроение, 1986. 207 с.
23. Zuev V.A., Gramuzov E.M., Dvoichenko Y.A., Sebin A.S. The Experimental and Theoretical Investigations of the “GP-ice” Use in Ships Model Tests // IOP Conference Series: Earth and Environmental Science. IOP Publishing, 2020. Vol. 459, N 4. P. 042071.
24. Давыдов В.В., Маттес Н.В., Сиверцев И.Н. Учебный справочник по прочности судов внутреннего плавания. М.: Речной транспорт, 1958. 755 с.
25. Папкович П.Ф. Труды по строительной механике корабля. В 4 т. Т. 3. Сложный изгиб стержней и изгиб пластин. Л.: Судпромгиз, 1962. 527 с.