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DIFFERENT APPROACHES TO NUMERICAL MODELING OF SEA WAVES

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Abstract

The main approaches in direct modeling of surface waves based on complete equations of dynamics of the inviscid liquid with a free surface are briefly considered. Most of the models are intended for study of the applied and engineering problems. It is assumed that the main model is written in the curvilinear coordinate system where the height is counted off from wave surface. In the two-dimensional periodic formulation, when using a conformal system, the problem is reduced to the system of one-dimensional equations that can be easily solved using Fourier-transform method. For three-dimensional waves such simplifications do not exist, thus, the vertical velocity on the surface is calculated by solving a three-dimensional Poisson equation or using a surface integral method. An approximate scheme based on the two-dimensional equations is considered. The scheme allows reproducing the statistical mode of waves with high accuracy consistent with the similar results obtained from the accurate three-dimensional model.

Keywords: simulation, wind waves, wave development, wave spectrum, Fourier-transform method, wave boundary layer, wind input, waves dissipation, wave statistics

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РАЗЛИЧНЫЕ ПОДХОДЫ К МОДЕЛИРОВАНИЮ МОРСКИХ ВОЛН

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Аннотация

Кратко рассмотрены главные подходы в прямом моделировании поверхностных волн, основанные на полных уравнениях динамики невязкой жидкости со свободной поверхностью. Большая часть моделей предназначена для исследования прикладных и инженерных проблем. Предполагается, что основной является модель, записанная в криволинейной системе координат, в которой высота отсчитывается от волновой поверхности. В двумерной периодической формулировке при использовании конформной системы задача сводится к системе одномерных уравнений, легко решаемых с использованием Фурье метода. Для трёхмерных волн такие упрощения не существуют, и вертикальная скорость на поверхности рассчитывается путём решения трёхмерного уравнения Пуассона или методом поверхностного интеграла. Рассматривается приближённая схема, основанная на двухмерных уравнениях. Схема позволяет воспроизводить статистический режим волн с высокой точностью согласующийся с аналогичными результатами, полученными по точной трёхмерной модели.

Ключевые слова: моделирование ветровых волн, развитие волн, волновой спектр, Фурье метод, вертикальная скорость на поверхности, приток энергии от ветра к волнам, диссипация волн, волновая статистика

1. Introduction

The numerical modeling in fluid mechanics can be defined as a method of simulation of the real process based on full nonlinear equations. The most popular method of the surface wave simulation is based on the spectral representation of wave field. This technology was developed specifically for wave forecasting. Currently, this approach is the only possible method that allows describing an evolution of wave field over the large areas for long periods. Despite

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a broad range of applicability, the spectral modeling cannot be referred to the numerical modeling since it is based on the assumption of linearity. In fact, the spectral description does not contain any information on waves, but only on distribution of wave energy over direction and frequency in each point of the domain. For example, those data cannot be used for calculation of probability of wave height. The spectral model assumes that wave field consists of superposition of linear waves with random phases and an arbitrary angle distribution. Being converted to a physical wave field, it looks unreal, because real waves usually have the prolonged smooth troughs and sharp peaks.

An alternative approach is based on the equations that are able to reproduce the movement of real waves. Such approach is sometimes called "the phaseresolving modeling" or "direct modeling" that is defined as a mathematical modeling of surface waves including an explicit simulation of surface elevation and a velocity field evolution. Compared with the spectral wave modeling, the phase resolving modeling is more general since it reproduces the real visible physical process and is based on the well-formulated full equations. The phase resolving models usually operate with a large number of degrees of freedom. Being applied to similar tasks, the direct method is more complicated and requires more computational resources. The simplest way of such modeling is calculation of a wave field evolution based on the linear equations. Such approach allows reproducing the main effects of the linear wave transformation due to superposition of wave modes, reflections, refraction etc. This approach is useful for many technical applications, although it cannot reproduce the nonlinear nature of waves and transformation of wave field due to nonlinearity. Another example of a relatively simple object is the case of shallow-water waves. The nonlinearity can be taken into account in the more sophisticated models derived from the fundamental fluid mechanics equations with some simplifications.

The most popular approach is based on a nonlinear Schrödinger equation of different orders [1] obtained by expansion of the *surface wave* displacement. The main advantage of a simplified approach is that it allows reducing of a three-dimensional (3-D) problem to a two-dimensional one (or 2-D problem to 1-D problem). However, it is not always clear which of the non-realistic effects are eliminated or included in the model after simplifications. For example, in Schrödinger equations the instability of high waves is missing; hence, the amplitudes of simulated waves can be unrealistically large. In reality, an excessive growth of waves is prevented due the wave-breaking instability. The most general approach being developed over the past years is based on the initial two-dimensional or three-dimensional equations (still potential).

All the tasks based on those equations can be divided into two groups: the periodic and non-periodic problems. An assumption of periodicity considerably simplifies construction of the numerical models though such formulation can be applied to the cases when the condition of periodicity is acceptable, for example, when the domain is considered as a small part of a large homogeneous area. For limited domains with no periodicity the problem becomes more complicated since the Fourier presentation cannot be used directly.

From the point of view of physics, a problem of phase resolving modeling can be divided into two groups: the adiabatic and non-adiabatic modeling. A simple adiabatic model assumes that the process develops with no input or output of energy. Being not completely free of limitations, such formulation allows investigating wave motion on the basis of true initial equations. Including the effects of dissipation is always connected with an assumption that generally contradicts the assumption of potentiality, i.e., the new terms added to the equations should be referred to as pure phenomenological.

All the phase resolving models use the methods of computational mathematics and inherit all their advantages and disadvantages, i.e., on one side, the possibility of a detailed description of the processes, on the other side, a bunch of specific problems connected with the computational stability, space and time resolution. The mathematical modeling produces huge volumes of information the processing of which can be more complicated than the modeling itself.

The phase resolving wave modeling takes a lot of computer time since it normally uses the surface-following coordinate system, which considerably complicates the equations. The most time-consuming part of the model is an elliptic equation for the velocity potential usually solved with iterations. Luckily, for a two-dimensional problem this obstacle is completely eliminated by use of the conformal coordinates reducing the problem to a one-dimensional system of equations which can be solved with high accuracy [2]. For a three-dimensional problem, the reduction to a two-dimensional form is evidently impossible; hence, solution of a 3-D elliptical equation for the velocity potential becomes an essential part of the entire problem. This equation is quite similar to the equation for pressure in a non-potential problem. It follows that the 3-D Euler equations being more complicated still can be solved over the acceptable computer time.

The main attention in the current review is devoted to the approaches related to the geophysical fluid dynamics. The models considered below are intended for investigation of mechanics of surface waves and development of parameterization of a physical process in the waves that can be used in the spectral wave forecasting models. The scope of the review is limited to the works devoted to free periodic waves and based on the principal equation for potential waves. The results obtained on the basis of the models are not a subject of the paper.

2. General approaches

In the past decades a large volume of papers devoted to the numerical methods developed for investigation of wave processes have been published. They include a Finite Difference Method [3, 4], a Finite Volume Method [5], a Finite Element Method [6, 7], a Boundary (Integral) Element Method [8], Spectral Methods [9–12] as well as a Smoothed Particle Hydrodynamics method [13]; a Large Eddy Simulation Method (LES) [14, 15], a Moving Particle Semi-implicit method [16]; a Constrained Interpolation Profile method [17], a Method of Fundamental Solutions [18] and a Meshless Local Petrov–Galerkin method [19]. The fully nonlinear model can be applied to many problems. Most of the models were designed for engineering applications such as the overturning waves at the beach, broken waves, the waves generated by landslides, freak waves, solitary waves, tsunamis, the violent sloshing waves, the interaction of extreme waves with beaches as well as the interaction of steep waves with fixed structures or different floating structures. The references given above make up less than one percent of the publications on those topics.

The problem of the numerical simulation of surface waves has a long history. The most general method to simulate motion with a free surface is based on some sort of the Lagrangian approach [20] which assumes the tracing of variable surface in a fixed grid with a different order accuracy (see, e.g., [21–24]). At present, the applicability of this method is restricted by simulation over the relatively short-term periods. However, the accuracy of this method will increase significantly when a very high resolution becomes possible. An advantage of this method is that it can be used for simulation of the 3-D rotational motion of a viscous fluid even for the non-single-value interface. The simulation of a nonlinear unsteady potential flow with a free surface began with development of the Eulerian–Lagrangian boundary integral equations approach [5] for steep overturning waves. This method, in principle, may be generalized for 3-D motion, but it requires considerable computational resources. The motion with the single-value 1-D and 2-D interface is easily simulated using the simplest surface-following coordinates where the height is counted off from the interface [26]. This system of coordinates is unsteady and non-orthogonal, so the equations of motion become complicated. The waves on finite depth were investigated by transforming the volume occupied by fluid into a rectangular domain [27]. The more complicated surface-following transformations have been constructed even for the case of a multiple-valued surface [28]. The grid method was generalized using adaptive grids (e.g., [29]) and a finite-volume approach [30].

3. Two-dimensional waves

Two-dimensional waves are a motion considered in a plane (x, z) (x is a horizontal coordinate and z is a vertical one). The numerous attempts to investigate such waves in the Cartesian coordinate system were not successful. In fact, the approach based on the non-stationary conformal mapping had been formulated even before it was used for numerical simulations. It had been introduced in [31, 32]), and later considered in [33–34]. Tanveer [35, 36] used that approach for investigation of Rayleigh–Taylor instability and generation of surface singularities. However, no authors of those works used conformal transformation for simulation of the long-term multi-mode periodic wave dynamics. Such 2-D model was completed in 1992, when a systematic use of the new approach to different problems was initiated. A numerical scheme based on conformal mapping (and its validation), as well as the results of the long-term simulations were presented in [37–39]. The scheme for arbitrary depth was described in [40, 41]. More details for a case of shallow water were given in [42]. Later, the method developed was used with some minor modifications in [43, 44] to demonstrate certain nonlinear properties of steep waves. The non-stationary conformal mapping for finite depth allows rewriting the principal equations of the potential flow with a free surface in the surface-following coordinate system. Laplace equation retains its form, while the boundaries of the flow domain (i.e., a free surface and, for the case of finite depth, the bottom) are the coordinate surfaces in a new coordinate system. Accordingly, the velocity potential in the entire domain receives a standard representation based on its Fourier expansion on a free surface. As a result, the hydrodynamic system of the equations (not simplified) is represented by two simple evolutionary equations which can be solved numerically in a straightforward way and used for theoretical investigations.

The assumption of potentiality simplifies the approach so significantly that the numerical scheme does not require any finite-difference approximations since the derivatives can be calculated precisely using the Fourier presentations, while the nonlinearities can be approximated on a dense grid with the well-estimated accuracy. If special measures (see [45]) are not taken, the calculations normally terminate much earlier due to the strong crest instability [25] shortly manifesting itself by separation of the falling volume into two phases. This phenomenon is obviously non-potential. Hence, as in many branches of geophysical fluid dynamics, some special measures (probably, precarious from the point of view of the potential theory) must be taken to prevent numerical instabilities, at the same time considering the physical consequences of such events.

The conformal model turns out to be a perfect tool for investigation of many problems of wave dynamics. The high accuracy of the model was proved by comparison of a nonstationary solution with the exact solutions for gravity waves (i.e. Stokes waves), gravity-capillary i.e. waves and capillary waves (i.e., [46]). The algorithms for calculations of stationary waves were developed in [41]. The most interesting feature of the multi-mode wave fields was a clear separation of the wavenumber-frequency spectra into regular curvilinear branches with most of the energy concentrated along what we call 'main branches' This effect was observed in [47]. The conformal modeling was used for investigation of many other problems of wave dynamics: the dynamics of extreme waves [48], statistical properties of one-dimensional waves [49, 50], Benjamin-Feir instability [51], freak waves [52, 53], wave breaking [54–56]. A systematic description of conformal modeling and the results obtained are given in a book [57].

The main advantage of conformal modeling is very high accuracy and performance. Even with a use of many thousands of Fourier modes, the long-term runs could be done for quite acceptable time. Hence, in many cases when the two-dimensionality is acceptable, the conformal model should be used instead of highly complicated three-dimensional models. The conformal model can be used for modeling and parameterization of a local physical process such as energy input to waves from wind, wave breaking, i.e. for the processes with relatively short time scales. The conformal model is evidently inapplicable for simulation of a developing wave field since the nonlinear interaction in 2-D space is not effective. A two-dimensional approach considers a strongly idealized wave field, since even monochromatic waves in the presence of lateral disturbances quickly obtain a two-dimensional structure. The difficulty arising is not a direct result of the dimension increase. The fundamental complication is that the problem cannot be reduced to a two-dimensional problem, and even for the case of a double-periodic wave field the problem of solution of Laplace equation for the velocity potential arises.

The majority of the models designed for investigation of three-dimensional wave dynamics are based on the simplified equations such as the second order perturbation methods in which the higher order terms are ignored. Overall, it is unclear which effects are missing in such simplified models.

4. Three-dimensional waves

Over the past decades many papers devoted to three-dimensional numerical methods developed for investigation of wave processes has been published. The three-dimensional modeling of surface waves based on full nonlinear equations is a powerful tool for investigation of wave processes, development of parameterization schemes for phaseresolving and spectral models and direct simulation of wave regimes in small basins. This type of modeling is rapidly developing. The most sophisticated method is based on full three-dimensional equations and surface integral formulations [58–65]. All models of such type extend an approach [66] originally given in the two-dimensional setting. The model is based upon a Hamiltonian formulation [67], which allows reducing a problem of surface variables computation by introducing a Dirichlet–Neumann operator which is expressed in terms of its Taylor series expansion in homogeneous powers of surface elevation. Each term in this Taylor series can be obtained from the recursion formula and efficiently computed using fast Fourier transforms.

The main advantage of the boundary integral equation methods (BIEM) is that they are accurate and can describe highly nonlinear waves. A method of solution of Laplace equation is based on the use of Green's function, which allows us to reduce a 3-D water-wave problem to a 2-D boundary integral problem.

The surface integral method is well suited for simulation of wave effects connected with very large steepness, specifically, for investigation of freak wave generation. These methods can be applied both to periodic and non-periodic flows. The methods do not impose any limitations on wave steepness, so they can be used even for simulation of waves that approach breaking [60], when the surface obtains a non-single value shape. The method allows us to take into account the bottom topography [68] and investigate the interaction of waves with fixed structures or with freely-responding floating structures [69, 70].

However, the BIEM method is quite complicated and time-consuming. It was used mostly for simulation of relatively simple wave fields, and it is unlikely that the method can be applied to simulation of a long-term evolution of the large-scale multi-mode wave field with a broad spectrum. The implementation of a multi-pole technique for the general problem of sea wave simulation [65] can solve the problem but obviously leads to the considerable algorithmic difficulties.

Currently, the most popular approach is a HOS (High Order Scheme) model developed in [71, 72]. The HOS is also based on the paper [67] where a convenient form of the dynamic and kinematic surface conditions was suggested:

The equations suggested by Zakharov were not intended for modeling, but for investigation of stability of the finite amplitude waves. In fact, a system of coordinates where depth is measured from the surface was used, but the Laplace equation for the velocity potential was taken in its traditional form. However, the Zakharov followers have accepted this idea literally. They used two coordinate systems: a curvilinear surface-fitting system for the surface

conditions and the Cartesian system for calculation of the surface vertical velocity. The analytical solution for the velocity potential in the Cartesian coordinate system is known. It is based on Fourier coefficients on a fixed level, while the true variables are Fourier coefficients for the potential on a free surface. Here a problem of transition from one coordinate system to another arises. This problem is solved by expansion of the surface potential in Taylor series in the vicinity of the surface. The accuracy of this method depends on estimation of an exponential function with the finite number terms of Taylor series. For the small-amplitude waves and for a narrow wave spectrum, such accuracy is satisfactory. However, for the case of a broad wave spectrum that contains many wave modes, an order of Taylor series should be high. The problem is now that the waves with high wave numbers are superimposed over the surface of larger waves. Since the amplitudes of the surface potential attenuate exponentially, the amplitude of a small wave at positive elevation increases, and on the contrary, can approach zero at negative elevations. It is clear that such formulation of HOS model cannot reproduce the high-frequency waves, which actually reduces the nonlinearity of the model. This is why such models can be integrated for long periods using no high frequency smoothing. In addition, the accuracy of calculation of a vertical velocity on the surface depends on full elevation at each point. Hence, the accuracy is not uniform along the wave profile. A substantial increase of an order of Taylor expansion can result in the numerical instability due to occasional amplification of the modes with high wave numbers. However, the HOS approach has been widely used (for example, [73–75]). It has shown its ability to efficiently simulate the wave evolution (propagation, nonlinear wave–wave interactions, etc.) in a large-scale domain [76–77]. It is obvious that HOS model can be used for many practical purposes. HOS-Ocean is a numerical model dedicated to propagation of the non-linear wave fields in the open ocean with an arbitrary constant depth in a double-periodic domain.

Another group of 3-D models use direct solution of Laplace equations written in the curvilinear coordinate system. The most universal approach based on the 3-D finite-difference model is being developed at the Technical University of Denmark (TUD model, see [4]). The model is intended for calculation of wave dynamics in a basin of arbitrary shape and topography, containing the fixed and floating bodies. Evidently, this model is targeted at solution of different engineering problems of designing and operation of marine technique. However, the model is so universal that is can be applied to simulation of wave regimes in small basins. The parameterization of physical processes in TUD model was not yet presented.

A model [78] developed at Institute of Oceanology RAS in association with Melbourne University is not quite universal, but it is supplied with the developed algorithms for parameterization of the physical processes. The system of equations includes the evolutionary kinematic and dynamic surface conditions and Laplace equation for the velocity potential. The double-periodic domain of infinite depth is considered. Construction of the exact numerical scheme for a long-term integration of these equations in the Cartesian coordinate system is impossible, since the surface moves between the grid knots. Instead, the system of the curvilinear coordinates fitted with a surface is introduced. The penalty follows immediately after turning the simple Cartesian coordinates into the curvilinear, nonstationary and non-orthogonal coordinate system. Fortunately, the evolutionary equations become just slightly complicated, while Laplace equation transforms into a full elliptic equation. At each time step this equation can be represented as Poisson equation with the right-hand side depending on the solution itself as well as on the metric coefficient. Since the norm of the right sight of the equation is usually small, the solution of Poisson equation can be found with a three-diagonal matrix algorithm and the iterations over the right-hand side. This procedure being formulated in the Fourier space is greatly simplified by the assumption of periodicity. The significant simplification of the problem can be achieved by separation of the velocity potential into the linear and nonlinear components. It is assumed that a linear component satisfies Laplace equation with the known solution. The equation for a nonlinear component can be obtained by extracting Laplace equation from the initial Poisson equation. Such procedure has a lot of advantages since a nonlinear component is on the average less by 1–2 decimal orders than the linear one. It means that for solution of the reduced Poisson equation a lesser number of levels in vertical, a lesser number of iterations and a smaller accuracy criterion can be used. The use of two components in the evolutionary equation does not seem to provide noticeable advantages, however this way deserves further consideration. The potential wave problem gives a unique opportunity for validation of the full nonlinear model in comparison with the exact stationary solution obtained in the conformal coordinates [41]. The structure of Stokes wave was supported within a long interval of integration. After implementation of the energy input scheme and wave-breaking parameterization, the model was used for direct simulations of a two-dimensional wave field evolution under the action of wind, nonlinear interactions and dissipation [79].

5. 2-D model for simulation of 3-D waves

All 3-D models based on full equations have a common limitation, i.e. a low computational efficiency. Working with such models, even with modest resolution, turns into the endless waiting of the results. This property of the mod-

els slows down their improvement, in particular, development of parameterization schemes for physical processes since such work requires multiple repetitions of the runs. Such schemes significantly depend on the model resolution; it limits the possibility of the research with low-resolution models.

Recently, an attempt to develop a new approach to the phase-resolving modeling of two-dimensional periodic wave fields was undertaken in [80, 81]. The main idea of the scheme follows from the presentation of the velocity potential as a sum of the linear and nonlinear components suggested in [78]. The solution for a linear component is known; hence, a nonlinear component should be calculated through Poisson equation with a zero boundary condition on the surface. Such approach offers a new way to simplify the calculation by considering 2-D Poisson equation on the surface. The equation that can be treated as the additional surface conditions contains both the first φ_ζ and second $\varphi_{\zeta\zeta}$ vertical derivatives of the potential. Thus, the system of equations remains unclosed. It was empirically discovered that those variables are closely connected to each other. The linear dependence between φ_ζ and $\varphi_{\zeta\zeta}$ was checked in [80, 81]. It was shown that the use of such hypotheses leads to formulation of the closed system of equations, which allows obtaining the results close to those obtained with the full wave model.

Thus, a new approach is based on the following features of mathematical formulation of the potential surface deep-water wave dynamics:

1. The evolution of wave field is well described by a linear subsystem of linear equations with the relatively small corrections due to nonlinearity. The time scale for the non-adiabatic and nonlinear transformation of spectrum exceeds by 3–4 decimal order the character period of waves.

2. The three-dimensional structure of waves is described on the basis of Laplace equation for the velocity potential written in the surface-fitted coordinate system where Laplace equation turns into a general elliptical equation that can be represented as Poisson equation with the right-hand side depending on solution. For solution of this equation more than 95 % time and memory is required.

3. The evolutionary equations for the surface potential include surface values of the vertical velocity calculated after the equation for the velocity potential has been solved. Other information obtained in the course of calculation of 3-D field of the surface potential is not used.

4. The surface vertical velocity can be represented as a sum of a linear component (described analytically) and a relatively small nonlinear correction.

5. Poisson equation written for surface gives an additional surface condition, containing both the vertical velocity and its vertical derivative. Typically, the nonlinear correction to the vertical velocity is by one order smaller than the full vertical velocity.

6. The vertical profiles of a nonlinear component of the surface potential in the vicinity of surface calculated by the 3-D full wave model (FWM) [79, 57] allow us to suggest that the vertical velocity and its vertical derivative can be simply connected locally in a physical space.

7. This connection has been investigated and parameterized on the basis of the exact FWM. That allows us to exclude a vertical derivative of vertical velocity and transform the entire 3-D formulation into 2-D (surface) formulation.

A detailed derivation of 2-D equation is given in [80–82]. Finally, the system of 2-D equations for simulation of a multi-mode wave field evolution includes standard evolutionary kinematic and dynamic surface conditions and an additional diagnostic condition for calculation of the surface vertical velocity [82]:

$$\eta_\tau = -\eta_\xi \varphi_\xi - \eta_\vartheta \varphi_\vartheta + (1+s)w, \tag{1}$$

$$\varphi_\tau = -\frac{1}{2}\left(\varphi_\xi^2 + \varphi_\vartheta^2 - (1+s)w^2\right) - \eta - p, \tag{2}$$

$$\tilde{w} = \frac{A\left(2\left(\eta_\xi w_\xi + \eta_\vartheta w_\vartheta\right) + \Delta\eta w - s\bar{w}_\zeta\right)}{1+s}. \tag{3}$$

Here ξ and ϑ are 'horizontal' surface-following coordinates; ζ is a vertical coordinate ($\zeta = 0$ on the surface and it grows upwards); τ is time; $\eta(\xi, \vartheta, \tau)$ is the surface elevation above a mean level; η_τ is a time derivative; η_ξ and η_ϑ are horizontal derivatives of η over ξ and ϑ; φ is the surface velocity potential; φ_ξ and φ_ϑ are the along-surface derivatives over ξ and ϑ; $\varphi_\xi = w$ is a vertical derivative of the potential on the surface (i.e. the surface vertical velocity); $\Delta = \eta_{\xi\xi} + \eta_{\vartheta\vartheta}$ is the 2-D 'horizontal' Laplacian of elevation; $s = \eta_\xi^2 + \eta_\vartheta^2$; p is the air pressure on the surface, divided by water density.

The vertical velocity w is represented as a sum of the linear $\bar{w}$ and nonlinear $\tilde{w}$ components. A linear component $\bar{w} = \bar{\varphi}_\zeta$ is calculated analytically since $\bar{\varphi}$ satisfies Laplace equation $\bar{\varphi}_{\xi\xi} + \bar{\varphi}_{\vartheta\vartheta} + \bar{\varphi}_{\zeta\zeta} = 0$ which is solved with the boundary conditions $\bar{\varphi}(\zeta = 0) = \varphi$ and $\bar{\varphi}_\zeta(\zeta \to \infty) \to 0$. The surface distribution of a nonlinear component is calculated from Eq. (3).

Coefficient A in (3) is calculated by the formula

$$A = \sigma F(\mu), \tag{4}$$

where σ is dispersion of elevation η, while μ is a parameter

$$\mu = \sigma\sigma_L, \tag{5}$$

where σ_L is the dispersion of the horizontal Laplacian of elevation $\Delta\eta$. The function F is approximated as follows,

$$F = \frac{d_0\mu + d_1}{\mu + d_2}, \tag{6}$$

where $d_0 = 0.535$, $d_1 = 0.0414$, $d_2 = 0.00321$. The function $F(\mu)$ is shown in fig. 2 in (Chalikov, 2021a).

The equations (1–3) are written in a nondimensional form, which formally follows from the suggestion that the acceleration of gravity is equal to one. Note that this form of equation is highly convenient because adiabatic equations are self-similar and the results can be easily transformed into a dimensional form by choosing a specific length scale and appropriate scales for variables [57]. Remarkably, a nondimensional form of equations is also reasonable to use in the non-adiabatic formulation, as transformation of the integral characteristics of solution occurs at the scales many times larger than the period and length of wave.

An approximation (6) is also correct for the dimensional variables, because it is independent of an external scale L. The form of relations (4, 5) and the constants in (6) were found on the basis of numerous numerical experiments with FWM and empirical (in a computational sense) selection of nondimensional variables as well as the function F and the numerical parameters. This is why that simplified model is named *Heuristic Wave model* (HWM).

The numerical scheme is based on Fourier-transform method with the use of Fourier and grid presentations of variables; moreover, the number of freedom degrees in the grid space exceeds four times that in Fourier space. The vertical velocity w is calculated with Eq. (3) by iterations. For integration in time, the forth-order Runge-Kutta scheme was used.

An equation (3) is represented in a form convenient for iterations. It is also solved with Fourier-transform method. Since the iterations start from the initial condition taken from a previous time step, the typical number of iterations is two and never exceeds four. The occurrence of the computational instability usually signals that the dissipation cannot cope with excessively large surface steepness or curvature.

The simplified model gives almost the same statistical results as the full model. It is easy to see that the equations suggested above are completely similar to complete equations, with one exception though: a small non-linear correction to the total vertical velocity is calculated not from 3-D Poisson equation but from a simple 2-D equation (3). It can be added that the simplified model has a much simpler structure than FWM and is easily programmed.

It is obvious that the approach developed here is not precise. It cannot be applied for the individual cases with a small number of modes, for example, for simulation of a steep Stokes wave, as it was demonstrated for FWM in [57, 79]. The model is intended for simulation of dynamics and statistics of the homogeneous multi-mode wave field.

6. Non-adiabatic terms

The full adiabatic model is suitable for the relatively short time intervals only. Real waves receive energy from wind and dissipate. Currently, such processes are poorly studied because they are evidently more complicated than the wave movement itself. Therefore, neither of the models can be considered as fully adequate. The algorithms describing the input energy from wind and wave dissipation have been developed for wave forecasting models (see, for example [83]), but this scheme is applicable for relatively low wave numbers.

The energy input term is calculated on the basis of Miles theory. According to this theory, the energy input in a small interval in Fourier space is proportional to the wave energy in this interval with a coefficient depending on the wave age (a ratio of the mean phase velocity in a spectral interval to the wind velocity). The shape of this coefficient (called β-function) is poorly known (see [84]).

A nonlinear flux of energy directed to small wave numbers produces downshifting of spectrum, while an opposite flux forms a shape of the spectral tail. The second process can produce accumulation of energy near a ‘cut’ wave number. Both processes become more intensive with increase of energy input. The growth of amplitudes at high wave numbers is followed by that of the local steepness and numerical instability. This well-known phenomenon in the numerical fluid mechanics is eliminated by use of a highly selective filter simulating the nonlinear viscosity.

The main process of wave dissipation is the wave breaking. The instability of interface leading to breaking is an important and poorly developed problem of fluid mechanics. In general, this essentially nonlinear process should be

investigated for a two-phase flow. Such approach was demonstrated, for example, in [85]. The problem of the breaking parameterization includes two points: (1) establishment of a criterion of the breaking onset and (2) development of an algorithm of the breaking parameterization. The problem of breaking is discussed in details in [86]. It was found in [87] that a clear predictor of breaking formulated in dynamical and geometrical terms, probably, does not exist. The most evident criterion of breaking is the breaking itself, i.e., the process when some part of the upper portion of a sharp wave crest is falling down. This process is usually followed by separation of the detached volume of liquid into the water and air phases. Unfortunately, there is no possibility to describe this process within the scope of the potential theory. The algorithm for the breaking parameterization must prevent numerical instability. The situation is similar to the numerical modeling of turbulence (LES technique) where the local highly selective viscosity is used to prevent appearance of too high local gradients of the velocity and pressure. The description of breaking in the direct wave modeling should satisfy the following conditions: (1) it should prevent the onset of instability at each point of millions of grid points over more than 100 thousand of time steps. (2) It should describe in a more or less realistic way the loss of the kinetic and potential energies with preservation of balance between them. (3) It should preserve the volume. It was suggested in [41] that an acceptable scheme could be based on a local highly selective diffusion operator with a special diffusion coefficient.

In general, the problem of parameterization of the physical processes is far from completion. All the schemes contain the coefficients which values are known with low accuracy. However, by tuning those coefficients it is possible to simulate wave development under the action of wind more or less plausibly.

7. Conclusion

At present, the direct wave modeling forms an independent section of the computational fluid mechanics. Most of the models are targeted at the technical problems such as wave dynamics of the coastal zone, the interaction of waves with various objects, the design and operation of marine technique. Nevertheless, direct modeling has made a significant contribution to the geophysical research, i.e. the mechanics and statistics of nonlinear sea waves (including the problems of extreme waves). The role of direct modeling will increase due to the fact that it can be used for development of the spectral modeling. Even now, the direct modeling can be used to study wave regimes in small reservoirs with the real bathymetry.

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