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Formulas for Estimating the Runup and the Force Exerted by a Bore and a Solitary Wave on a Partially Immersed and Fixed Structure, Obtained by Approximating the Results of Numerical Modeling

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Abstract

This study derives analytical formulas for estimating the loads exerted by a bore and a solitary wave on a fixed, partially immersed body. These estimates are required in the design and operation of coastal structures. The formulas are obtained by fitting results from extensive numerical simulations spanning variations in body submergence, body length, and incident-wave amplitude. We consider runup heights on the front and back sides of the body, as well as the horizontal and vertical components of the resultant wave force. The bore and solitary-wave problems are solved using one-dimensional shallow water models based on the first and second long-wave approximations, respectively. We report the average and maximum relative errors of the formulas and compare their predictions with results from previous studies. These comparisons demonstrate that the proposed formulas are applicable across the parameter ranges considered.

Keywords: solitary wave, vertical bore, partially immersed structure, shallow water, nonlinear dispersion, numerical algorithm, data fitting

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Формулы для оценки воздействия бора и уединённой волны на полупогруженное тело, полученные аппроксимацией результатов численного моделирования

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Аннотация

Настоящая работа посвящена построению аналитических соотношений для оценки характеристик воздействия вертикального бора и уединённой волны на полупогруженное фиксированное тело. Такие оценки необходимы при проектировании и эксплуатации объектов, размещённых и зафиксированных в прибрежных зонах в виде частично погруженных в воду сооружений. Эти соотношения строятся при помощи аппроксимации результатов выполненных массовых расчётов с перебором таких параметров задачи, как заглубление и длина тела, амплитуда набегающей волны.

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Рассматриваются максимальные заплески на лицевую и тыльную грани тела, горизонтальная и вертикальная составляющие суммарной волновой силы. Задачи о воздействии бора и уединённой волны численно решаются с использованием одномерных моделей мелкой воды первого и второго длинноволнового приближения соответственно. Приводятся оценки средней и максимальной относительных погрешностей формул, а также сопоставления получаемых с их помощью результатов с решениями из других исследований.

Анализ этих сопоставлений позволяет сделать вывод о возможности применения построенных формул в рассматриваемом диапазоне параметров задачи.

Ключевые слова: уединенная волна, вертикальный бор, полупогруженное тело, мелкая вода, нелинейная дисперсия, численный алгоритм, аппроксимация данных

1. Introduction

When designing and operating fixed structures deployed in coastal zones — such as liquefied natural-gas storage facilities or floating nuclear power plants — it is essential to assess the loads exerted by surface waves, including long-period waves of tsunami type. These problems have been addressed analytically [1, 2] and experimentally [3–5], but in recent years an increasing number of studies have focused on numerical modelling [6, 7]. A large fraction of recent studies focus on isolated test cases of wave impact on a partially immersed body [8–10] to demonstrate the performance of a proposed numerical scheme. In other studies [11–13], a limited set of parameters is fixed, and the dependence of the resulting wave-impact characteristics on a single variable is illustrated graphically.

For engineers and designers, these results are of limited practical use, and only a few studies provide usable formulas for estimating wave-impact characteristics on a partially immersed body, even in the simplest formulation. Based on laboratory experiments and numerical simulations using the Navier–Stokes equations, Liu and Al-Banaa [14] proposed formulas for the maximum runup of a solitary wave on the front (closest to the initial wave position) side of a body and for the horizontal component of the resultant wave force, but only for a single, very small body length. Using analytical methods, Dorfman et al. [15] derived a formula for the horizontal component of the wave force exerted by a vertical bore on a partially immersed body.

At the 17th All-Russian Conference *Advanced Technologies in Hydroacoustics and Hydrophysics*, the authors [16] presented numerical results on the interaction of long surface waves with large, fixed, partially immersed coastal structures. Here, we examine in greater detail the construction of analytical expressions for estimating the loads exerted by a vertical bore and a solitary wave on a partially immersed fixed body. We consider the most simplified configuration, in which the interaction is assumed to be independent of one horizontal coordinate, the seabed and the body's underside are taken as horizontal, and the body's side faces are vertical.

The core idea is to compute a large set of bore and solitary-wave impact scenarios while systematically varying the governing parameters within certain limits — the initial wave amplitude a_0 , the submergence d and the length L of the body — and then to approximate the resulting maximum runup on the front and back faces of the body, as well as the horizontal and vertical components of the wave-induced force, using formulas suitable for engineering applications. The bore impact is examined with the classical shallow water (SV) model [17], which preserves the bore shape during propagation, whereas the solitary-wave impact is treated within the nonlinear dispersive Serre–Green–Naghdi (SGN) framework for the same reason: the solitary wave also propagates without deformation. Because several thousand simulations are required, the computational efficiency afforded by shallow water models is essential.

We adopt the following procedure to construct the formulas that approximate the numerical results:

1. Define parameter ranges: Identify the ranges of the governing parameters for which the adopted models remain accurate, and generate a discrete set of values spanning these ranges.
2. Perform simulations: Carry out the full series of calculations and record the required wave-impact characteristics in structured output files.
3. Analyse single-parameter dependencies: Examine how each characteristic depends on each parameter individually — for example, the dependence of the maximum runup on the front face on the wave amplitude while holding the other parameters fixed — and select functional forms that reproduce these trends adequately.
4. Build a composite formula: Combine the single-parameter dependencies with available a priori information about the characteristic to construct a general expression, potentially containing many coefficients.
5. Optimize coefficients: Determine the “optimal” set of coefficients by minimizing the discrepancy between the formula and the numerical results.

6. Simplify the expression: Reduce the formula by eliminating the least significant terms, reoptimizing the coefficients after each reduction (as in Step 5).

Section 2 provides a brief overview of the shallow water models used in the simulations — the second long-wave approximation (SGN) and the first long-wave approximation (SV) [18] — and outlines the methodology for constructing formulas that approximate the numerical results. Section 3 presents the derivation of analytical relationships for estimating the characteristics of bore impact, whereas Section 4 addresses the corresponding problem for a solitary wave. Finally, Subsection 4.5 illustrates the application of the derived formulas and compares the resulting estimates with the calculations of Ma et al. [10].

2. Models and methods

2.1. Problem statement for shallow water models

We consider a free-surface flow in a basin bounded by impermeable, vertical sidewalls and containing a fixed, partially immersed rectangular body with a horizontal underside. The surface waves are assumed to propagate normal to the body's side faces, and all flow variables are taken to be independent of one horizontal coordinate. A Cartesian coordinate system Oxy is introduced with vertical axis Oy , such that the undisturbed free surface lies at $y = 0$, and the horizontal basin floor is defined by the equation $y = -h_0$ (Fig. 1). In this case, the left and right boundaries of the basin are located at $x = 0$ and $x = l$, respectively, the front and back faces of the body are at $x = x_l$ and $x = x_r$ ($0 < x_l < x_r < l$), and the underside of the body is defined by $y = -d$ ($0 < d < h_0$).

The interaction of a surface wave with a partially immersed body is examined using shallow water models of the first (SV) and second (SGN) long-wave approximations for an incompressible, inviscid fluid of density ρ (hereafter $\rho = 1$). The computational domain $D = (0, l)$ is divided into an outer subdomain $D^e = (0, x_l) \cup (x_r, l)$, where the flow has a free surface, and an inner subdomain $D^i = (x_l, x_r)$ where the flow occurs in the gap between the body's underside and the basin bottom. To reconcile the solutions in these subdomains, we impose compatibility conditions at their common boundaries.

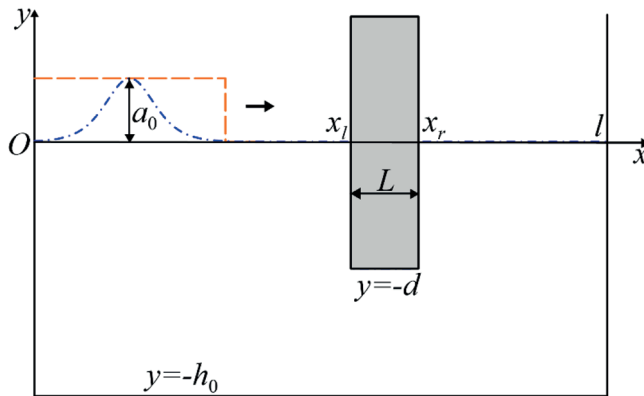


Fig. 1. Schematic diagram of the computational domain in the problem of interaction of the bore (dashed line) and the solitary wave (dash-dotted line) with a partially immersed and fixed structure of rectangular shape

2.2. Equations in the outer subdomain D^e

The equations of the nonlinear dispersive SGN model in D^e [18] for the case of a horizontal basin floor can be expressed as

$$H_t + (Hu)_x = 0, \quad (Hu)_t + (Hu^2 + p)_x = 0, \quad x \in D^e, \quad (1)$$

where t is time, $u(x, t)$ is the depth-averaged horizontal flow velocity, $H(x, t) = \eta(x, t) + h_0$ is the total depth, $\eta(x, t)$ is the deviation of the free surface from the undisturbed level, p is the main part of the pressure held in the long-wave approximation, $p = g \frac{H^2}{2} - \varphi$, g is the constant gravity acceleration, φ is the dispersive component of p , $\varphi = \frac{H^3}{3} (u_{xt} + uu_{xx} - u_x^2)$.

The primary unknowns in these equations are the total depth H , the velocity u and the pressure dispersive component φ . The latter is determined as the solution of a second-order ordinary differential equation that is linear in φ :

$$(k\varphi_x)_x - k_0\varphi = F, \quad (2)$$

where $k = 1/H$, $k_0 = 3/H^3$, $F = g\eta_{xx} + 2u_x^2$.

The boundary conditions for the system of equations (1), (2) are specified at the boundary of the basin $\Gamma_0 = \{0; l\}$ [18] and at the common boundary $\Gamma = \{x_l; x_r\}$ between the subdomains D^e and D^i : $\eta_x|_{x \in \Gamma} = 0$, along with the compatibility conditions at $x \in \Gamma$ (see Subsection 2.4) and the initial conditions at D . The SV model equations in the D^e region are derived by neglecting all terms containing the dispersive component φ .

The right-hand boundary of the domain is placed sufficiently far from the body so that waves reflected from it do not influence the wave-induced characteristics at the body.

2.3. Equations in the inner subdomain D^i

The system of equations governing unsteady intrachannel flow of an ideal, incompressible fluid is considered for the inner subdomain [19]. For the one-dimensional problem examined here, with a fixed horizontal underside of the body and basin floor, this system can be expressed as:

$$u_x^i = 0, \quad u_t^i + p_x^i/S_0 = 0, \quad x \in D^i \quad (3)$$

where S_0 is the constant thickness of the fluid layer under the body, $S_0 = h_0 - d$, $u^i(x, t)$ is the horizontal component of the flow velocity averaged over S_0 , $p^i(x, t)$ is the pressure integrated over S_0 .

The first equation in (3) expresses the independence of the velocity from the coordinate x , which is consistent with incompressible flow in a channel of constant cross-section. Consequently, the velocity beneath the body depends only on time t , and in this region, the pressure $p^i(x, t)$ and the flow rate $u^i(t)$ must be determined. The pressure can be excluded from the calculations by integrating the equation of motion in (3) over the inner subdomain D^i which yield the equation

$$\dot{u}^i(t) = -\frac{p|_{x_r-0} - p|_{x_l+0}}{S_0 L}. \quad (4)$$

In this equation and in the following, the notations $\cdot|_{x_l-0}$, $\cdot|_{x_r+0}$ denote the values at the boundaries of the outer subdomain D^e at x_l and x_r , respectively, while $\cdot|_{x_l+0}$, $\cdot|_{x_r-0}$ denote the values at the inner boundaries of the subdomain D^i . The equation (4) shows that changes in the fluid velocity beneath the body are driven by the pressure difference at the Γ boundary of the inner region D^i . These boundary pressure values p_i are determined using **compatibility conditions** that link the flows in the outer and inner subdomains.

2.4. Compatibility conditions

The mass flow condition of Khakimzyanov and Dutykh [19] for the one-dimensional problem considered here can be expressed as

$$(Hu)|_{x_l-0} = S_0 u^i = (Hu)|_{x_r+0} \quad (5)$$

and ensures that the flow of incompressible fluid entering beneath the body equals the flow leaving beneath the body, corresponding to the mass flow $S_0 u^i(t)$ under the body. The pressure relations [19] outside and under the body are defined in addition to (5). For the one-dimensional case with a horizontal basin floor and horizontal underside of the body, these relations for the SGN model can be written as

$$\frac{p^i|_{x_l+0}}{S_0} = \left[g \left(H - \frac{S_0}{2} \right) - \frac{\varphi}{2H^3} (3H^2 - S_0^2) \right]_{x_l-0}, \quad (6)$$

and the same for x_r . These relations remain valid for the SV model after the dispersive terms containing φ are omitted. It should be noted that this approach accounts for both waves reflected from the front side of the structure and waves that pass beneath the body and affect its back side.

2.5. Calculation of the components of the total wave force

For the SGN model, the horizontal component $F_h(t)$ of the hydrodynamic wave force on a rectangular body in a basin with a flat basin floor is determined using the formula [20]

$$\frac{F_h(t)}{\rho g h_0^2} = \left[\frac{(\bar{H} - \bar{S}_0)^2}{2} \left(1 - \frac{2\bar{H} + \bar{S}_0}{\bar{H}^3} \bar{\varphi} \right) \right]_{x_r+0}^{x_l-0}, \quad (7)$$

and for the SV model it can be simplified:

$$\frac{F_h(t)}{\rho g h_0^2} = (\bar{\eta}(x_l, t) - \bar{\eta}(x_r, t)) \left(\frac{\bar{\eta}(x_l, t) + \bar{\eta}(x_r, t)}{2} + \bar{d} \right). \quad (8)$$

Hereinafter, the formulas are given using dimensionless quantities $\bar{H} = H/h_0$, $\bar{\eta} = \eta/h_0$, $\bar{S}_0 = S_0/h_0$, $\bar{d} = d/h_0$, $\bar{L} = L/h_0$, $\bar{\varphi} = \varphi/(gh_0^2)$, $\bar{a} = a_0/h_0$. For the SGN model, the vertical component of the hydrodynamic force in the case considered is calculated using the formula [20]

$$\frac{F_v(t)}{\rho g h_0^2} = \frac{\bar{L}}{2} \left(\left(\bar{\eta} - \bar{\varphi} \frac{3\bar{H}^2 - \bar{S}_0}{2\bar{H}^3} \right) \right)_{x_l-0} + \left(\bar{\eta} - \bar{\varphi} \frac{3\bar{H}^2 - \bar{S}_0}{2\bar{H}^3} \right)_{x_r+0}, \quad (9)$$

and for the SV model

$$\frac{F_v(t)}{\rho g h_0^2} = \frac{\bar{L}}{2} (\bar{\eta}(x_l, t) + \bar{\eta}(x_r, t)). \quad (10)$$

2.6. Numerical method

The predictor-corrector scheme described in the monograph [21] and later modified in [17, 20, 22] is employed for the numerical solution within the shallow water models, including the numerical implementation of the compatibility conditions at the common boundary between the outer and inner subdomains. These models have been validated against the “reference” potential-flow model for an ideal fluid with a free surface (Pot model) [13] and against small-scale laboratory experiments by Lu and Wang [4]. We emphasize that using algorithms based on shallow water models allows a single scenario to be computed in only a few tens of seconds on a laptop, which is one to two orders of magnitude faster than the Pot model. This computational efficiency is particularly critical when simulating thousands of scenarios.

2.7. Data approximation

This subsection describes the methods and procedures used to approximate the numerical results with a formula of a given structure, in which the problem reduces to determining the coefficients that minimize the approximation error.

Minimizing the sum of squares of the absolute errors can be problematic when the target characteristic varies by several orders of magnitude across the parameter space. To avoid this issue, we instead minimize the sum of squares of the relative errors.

The Python SciPy Optimize library [23] procedures “least_squares” and “basinhopping” are employed to identify the optimal sets of coefficients. The “least_squares” procedure is used to solve nonlinear problems — in this case, with respect to the formula coefficients — by seeking a local minimum of the sum of squares of the relative errors.

The “least_squares” procedure is relatively fast and, although it does not guarantee a global minimum nor minimize the maximum relative error, it provides satisfactory results in many cases. The “basinhopping” procedure is somewhat slower but enables the identification of a global minimum for a given objective function. In this study, the objective function is defined as the product of the sum of squares of the relative errors and the maximum relative error.

3. Impact of a Bore on a Partially Immersed Structure

Gusev et al. [24] noted that most characteristics of the bore impact are independent of the body length L (similar conclusions were drawn by Dorfman et al. [15]), with the exception of the vertical force, which varies proportionally with L . The numerical algorithm employed in the present study was validated in [24] through comparison with the analytical solution for the bore impact on a vertical wall. In addition, the dependences of the horizontal and vertical components of the wave force on the body submergence were presented for a fixed initial wave amplitude $\bar{a} = 0.2$. In this section, we generalize these results and propose formulas that approximate the outcomes of the numerical calculations.

Series of calculations were performed for the following sets of parameters (the value of $\bar{L} = 5$ was kept constant in all cases):

$$\bar{a} = [0.01, 0.02, \dots, 0.1, 0.15, \dots, 0.5], \quad \bar{d} = [0.05, 0.1, \dots, 1]. \quad (11)$$

It should be noted that, to capture the discontinuous solutions with sufficient accuracy, the bore–structure interaction was computed using a very fine grid spacing, $\Delta x/h_0 = 0.005$.

3.1. Formula for estimating the maximum runup of the bore

We consider only the maximum runup on the front side of the body, since on the back side the free-surface elevation increases monotonically up to the initial bore amplitude. It was observed that the maximum bore runup is independent of both the body submergence d and its length L , while its dependence on the initial wave amplitude a_0 is well described by a quadratic function

$$R_l/h_0 = 0.423\bar{a}^2 + 2\bar{a}. \quad (12)$$

This formula has a maximum relative error of 0.7 % and an average relative error of 0.4 %.

Since this formula is independent of d , it can also be used to estimate the maximum bore runup on a vertical wall (case $\bar{d} = 1$) for which the analytical solution [25]. This solution is calculated for a given initial bore amplitude $\bar{a}$ by solving the nonlinear equation

$$f(z) = z^3 + 2(1 + \bar{a})z^2 - \bar{a}^2(2 + \bar{a})z - \bar{a}^2(2 + \bar{a})(1 + \bar{a}) = 0. \quad (13)$$

This equation has a single positive root z at each $\bar{a} > 0$ and the maximum runup is then calculated as $R_l/h_0 = z + \bar{a}$. The solutions obtained in this manner are virtually indistinguishable from those predicted by formula (12) across the entire range of initial bore amplitudes considered, confirming the validity of formula (12).

3.2. Formula for estimating the horizontal component of the bore impact

When constructing the formula for the horizontal component of the bore force, we take into account the following observations: the force is independent of L ; in slices, the dependence on d is well approximated by a linear function; in slices, the dependence on a_0 is well approximated by a cubic function; the force is zero when $a_0 = 0$.

The resulting formula

$$\frac{F_h}{\rho g h_0^2} = 1.987(\bar{a}^2 + \bar{a}\bar{d}) + 0.503\bar{a}^2\bar{d} + 0.859\bar{a}^3, \quad (14)$$

obtained by calculating the optimal coefficients, provides an excellent approximation of our numerical simulations, with a maximum relative error below 0.7 % and an average relative error of 0.2 %.

We also compare this formula with the engineering formula previously proposed by Dorfman et al. [15] for estimating the horizontal component of the wave force exerted by a bore on a partially immersed body:

$$\frac{F_h(t)}{\rho g h_0^2} = \frac{1}{2} \left(\left(\frac{H_3}{h_0} \right)^2 - 1 \right) \cdot \left(1 - \frac{(1 - \bar{d})}{H_3/h_0} \right), \quad (15)$$

where H_3 is the maximum total depth at the vertical wall, i. e. in the notations of this paper $H_3/h_0 = 1 + R_l/h_0$. When constructing formula (15), the force on the vertical wall is calculated first, and the force on the body

is then determined as a linear function of submergence, representing a fraction of the force on the wall. The value of H_3 is determined analytically, for example, from equation (13): at $\bar{a} = 0.01$ we have $H_3/h_0 = 1.02$; at $\bar{a} = 0.05$ — $H_3/h_0 = 1.10122$; at $\bar{a} = 0.2$ — $H_3/h_0 = 1.41833$; at $\bar{a} = 0.5$ — $H_3/h_0 = 2.10405$.

Figure 2 shows the horizontal component of the wave force calculated using formulas (14) and (15) for initial bore amplitudes $\bar{a} = 0.01, 0.05, 0.2$ and 0.5 . The comparisons indicate that for $\bar{d} = 1$, i. e. for the case of the impact on a vertical wall, the curves are in close agreement. However, the differences between them increase when $\bar{d}$ decreases, and, as a rule, formula (15) tends to overestimate the force values.

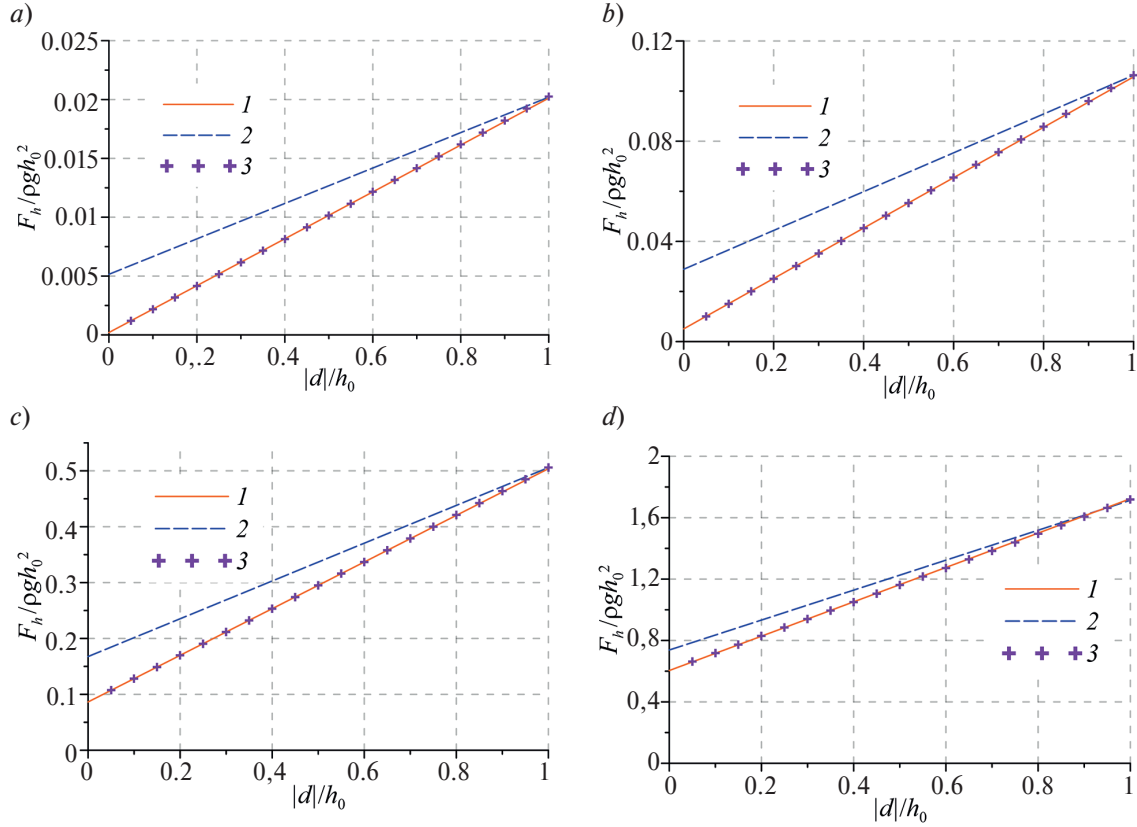


Fig. 2. Dependences on the submergence d of the maximum horizontal component of the total wave force exerted on the partially immersed body by the bore with initial amplitude $\bar{a} = 0.01$ (a), $\bar{a} = 0.05$ (b), $\bar{a} = 0.2$ (c) and $\bar{a} = 0.5$ (d). The graphs (1) and (2) are plotted using formulas (14) and (15) respectively, and the markers (3) correspond to the results of the numerical simulations

3.3. Formula for estimating the vertical component of the bore impact

The vertical component of the bore force is directly proportional to the body length L but independent of the initial wave amplitude d . The formula for calculating it is correspondingly simpler:

$$\frac{F_v}{\rho gh_0^2} = \bar{L} (0.286\bar{a}^2 + \bar{a}). \quad (16)$$

The maximum relative error of the formula is below 0.6 %, with an average relative error of 0.2 %.

4. Impact of a Solitary Wave on a Partially Immersed Structure

The interaction of a solitary wave with a partially immersed body was simulated using the SGN model, which has been validated against both the Pot model [13] and small-scale laboratory experiments [4]. As shown, for example, in [22], the maximum runup on the front and back sides of the body, as well as the amplitudes of the reflected wave and the wave passing beneath the body, are in good agreement between SGN and Pot model

calculations across a wide range of parameters. Exceptions occur for bodies of small length ($\bar{L} < 3$), where the differences can exceed 15 %. Similar conclusions can be drawn from the horizontal component of the wave force (Fig. 3).

Figure 3 allows a direct comparison between the Pot model [13] and SGN model results. When varying the body submergence d the agreement between models remains very good over the entire range. As the solitary wave amplitude a_0 increases, discrepancies grow but remain below 10 %. When the body length L decreases, differences increase, reaching more than 15 % for $\bar{L} < 3$, with the SGN model underestimating the strength values. Therefore, we strongly recommend that the SGN-based results presented here be applied only to problems with $\bar{L} \geq 3$.

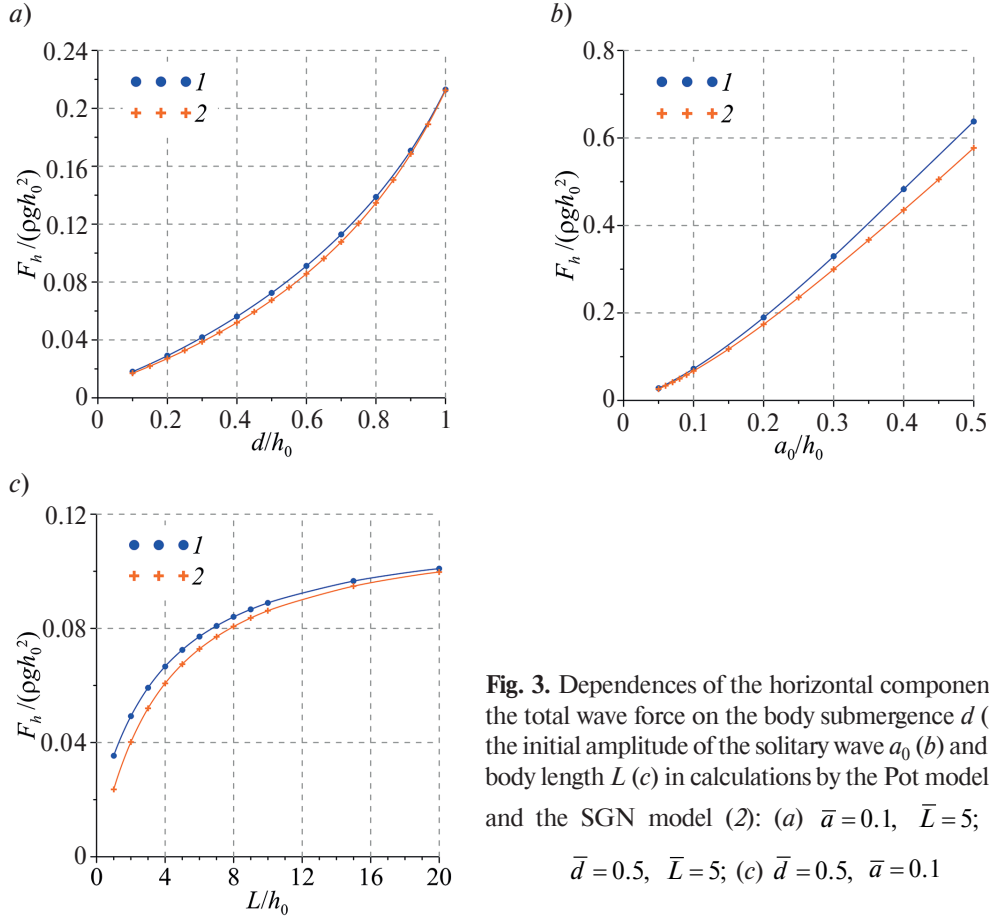


Fig. 3. Dependences of the horizontal component of the total wave force on the body submergence d (a), the initial amplitude of the solitary wave a_0 (b) and the body length L (c) in calculations by the Pot model (1) and the SGN model (2): (a) $\bar{a} = 0.1$, $\bar{L} = 5$; (b) $\bar{d} = 0.5$, $\bar{L} = 5$; (c) $\bar{d} = 0.5$, $\bar{a} = 0.1$

In total, 3240 numerical simulations of the interaction between a solitary wave and a partially immersed body were performed using the SGN model, with parameters varied over the following ranges:

$$\bar{a} = [0.01, 0.02, \dots, 0.1, 0.15, \dots, 0.5], \quad \bar{d} = [0.05, 0.1, \dots, 1], \quad \bar{L} = [3, 4, \dots, 10, 25]. \quad (17)$$

When preparing the final dataset for constructing the target formulas, cases with large initial solitary wave amplitudes and small body submergence — where the underside of the body became dry — were excluded, resulting in 3055 ‘acceptable’ cases.

4.1. Formulas for estimating the runup of the solitary wave on the front side of the body

To construct a formula approximating the maximum runup on the front side of a partially immersed, fixed body, we make the following assumptions: each of the dependences on the submergence d and the length L are well approximated by fractional linear functions; the dependence on the initial amplitude of the incident wave a_0 is well approximated by fractional-rational, where the degree of the numerator is equal to 3 and that of the denominator is equal to 1; the runup is zero when $a_0 = 0$; the runup is dependent of L for $\bar{d} = 1$.

The structure of the resulting “complete” formula can thus be expressed as follows:

$$\frac{R_l}{h_0} = \bar{a} \frac{\bar{L} (k_1 \bar{a}^2 \bar{d} + k_2 \bar{a}^2 + k_3 \bar{a} \bar{d} + k_4 \bar{a} + k_5 \bar{d} + k_6) + (1 - \bar{d}) (\bar{a}^2 + k_7 \bar{a} + k_8)}{\bar{L} (k_9 \bar{a} \bar{d} + k_{10} \bar{a} + k_{11} \bar{d} + k_{12}) + (1 - \bar{d}) (k_{13} \bar{a} + k_{14})}. \quad (18)$$

The procedure for finding the “optimal” values of the coefficients k_i yields the set presented in Table 1.

Table 1

The «optimal» values of the coefficients for formula (18)

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}
−0.078	0.26	−0.037	0.75	0.0031	0.0044	0.048	0.16	−0.032	0.39	0.0022	0.0016	0.42	0.17

Formula (18) with the coefficients listed in Table 1, provides a good approximation of the numerical results, yielding a maximum relative error of 2.5 % and an average relative error of 0.3 % (see line 2 in Fig. 4). The presence of coefficients in Table 1 that differ by several orders of magnitude motivated an attempt to simplify the formula by setting relatively small coefficients to zero, followed by a repetition of the optimization process. This yielded the following simplified formula:

$$\frac{R_l}{h_0} = \bar{a} \frac{\bar{L} \bar{a} (0.211 \bar{a} + 0.629) + (1 - \bar{d}) (\bar{a}^2 - 0.19 \bar{a} + 0.0822)}{0.32 \bar{L} \bar{a} + (1 - \bar{d}) (0.389 \bar{a} + 0.0822)}. \quad (19)$$

The maximum relative error of the formula is 3.7 %, with an average relative error of 0.9 % (see line 3 in Fig. 4).

In the case of a vertical wall ($\bar{d} = 1$), it follows from formula (19) that

$$\frac{R_l}{h_0} = 0.66 \bar{a}^2 + 1.966 \bar{a}. \quad (20)$$

The interaction of a solitary wave with a vertical wall has been extensively studied analytically, experimentally, and numerically. For instance, Byatt-Smith [26] derived the analytical expression

$$\frac{R_l}{h_0} = 0.5 \bar{a}^2 + 2 \bar{a}, \quad (21)$$

which was later refined by Su and Mirie [27]:

$$\frac{R_l}{h_0} = 2 \bar{a} + 0.5 \bar{a}^2 + 0.75 \bar{a}^3. \quad (22)$$

Based on laboratory experiments and numerical calculations, Liu and Al-Banaa [14] proposed a formula for estimating the runup of a solitary wave on a partially immersed body of very small width $\bar{L} \approx 0.16$, which in the case of $\bar{d} = 1$ is

$$\frac{R_l}{h_0} = 2.55 \bar{a}^{1.17}. \quad (23)$$

Figure 5 presents the dependence of the maximum runup on a vertical wall, calculated using formulas (20) — (23), alongside experimental data from [28–31] and results from the Euler model [32, 33] and the potential flow model [13]. Analysis of these graphs indicates that the various results are in good agreement up to wave amplitudes $\bar{a} = 0.5$, confirming the validity of formula (20) within this range and providing strong circumstantial support for the use of formula (19) to estimate the runup on the front side of a partially immersed, fixed body.

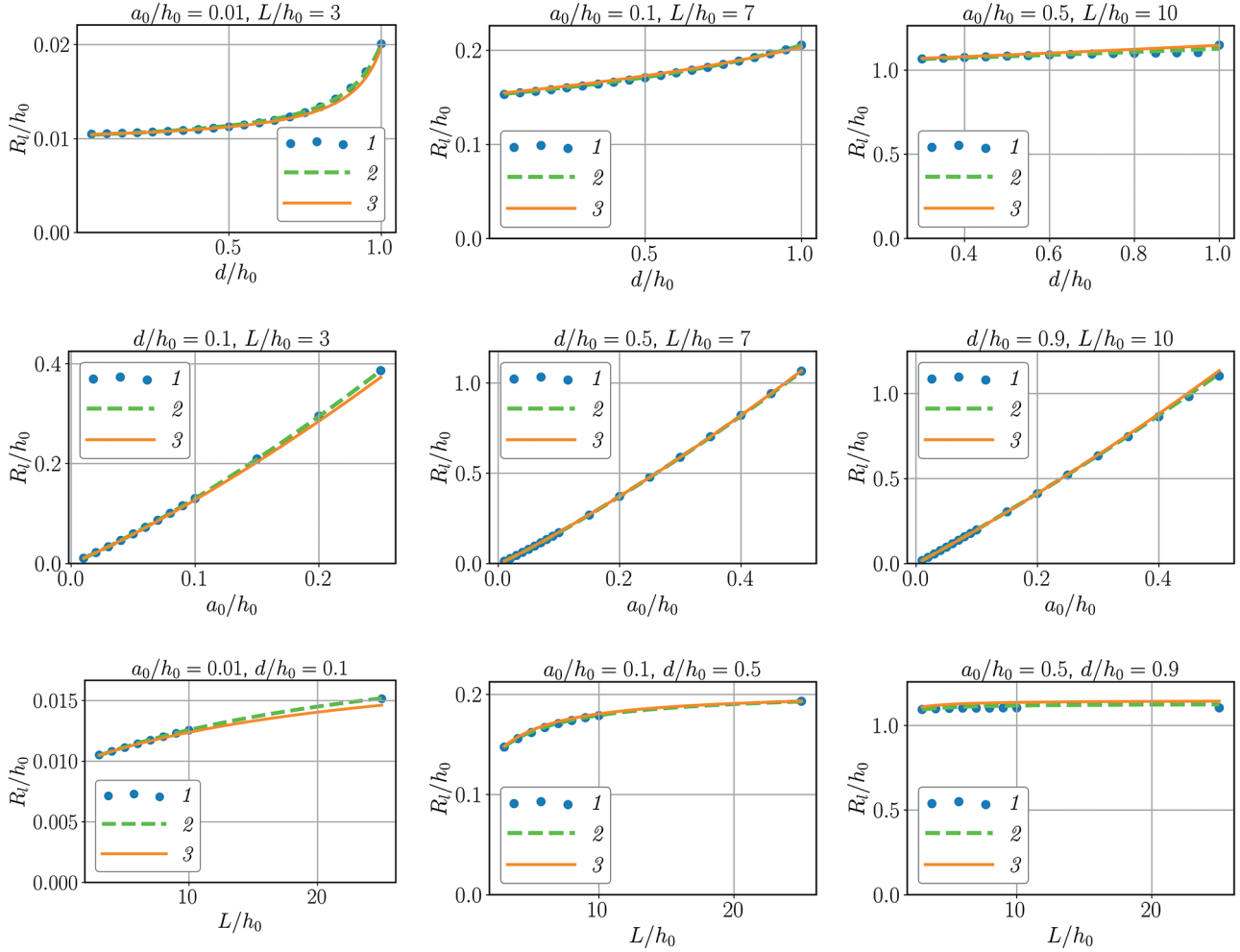


Fig. 4. Dependences of the maximum runup on the front side of the body on the body submergence d , the initial amplitude of the solitary wave a_0 and the body length L : 1 — the numerical solutions; 2 — formula (18); 3 — (19)

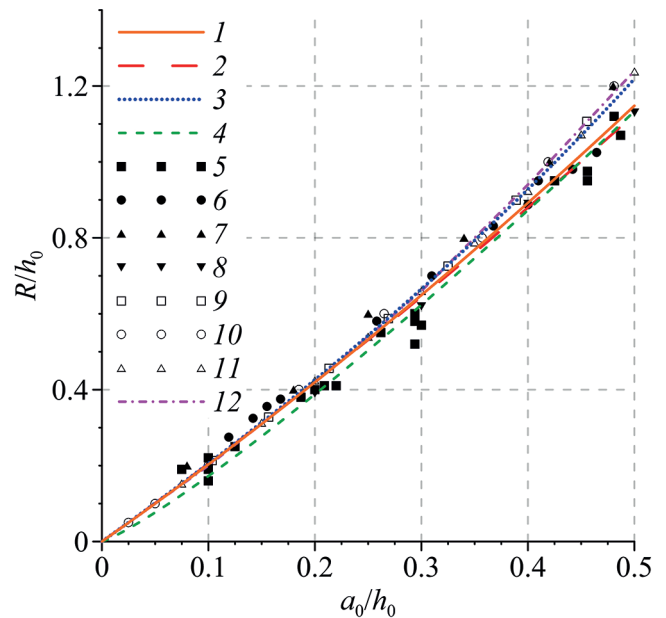


Fig. 5. Dependences of the maximum runup R on the vertical wall on the amplitude of the solitary wave a_0 obtained by formulas (20) — 1, (21) — 2, (22) — 3 and (23) — 4; according to laboratory experiments by 5 — [29], 6 — [28], 7 — [31], 8 — [30]; calculation results by 9 — [34], 10 — [32], 11 — [33], 12 — [13]

4.2. Formulas for estimating the runup of the solitary wave on the back side of the body

To construct a formula for estimating the maximum runup on the back side of a partially immersed, fixed body, we make the following assumptions: the dependence on the body length L is well approximated by a fractional linear function; the dependence on the submergence d is well approximated by a fractional rational function with degree 2 in the numerator and 1 in the denominator; the dependence on the initial wave amplitude a_0 is well approximated by a fractional rational function with degree 3 in the numerator and 1 in the denominator; the runup is zero at $a_0 = 0$; the runup is also zero at $\bar{d} = 1$ (the case of a vertical wall). Then the “complete” formula can be written as

$$\frac{R_r}{h_0} = \bar{a}(1 - \bar{d}) \left(\frac{N_1 + N_2}{D_1} \right), \quad (24)$$

where

$$\begin{cases} N_1 = k_1 \bar{a}^2 \bar{d} \bar{L} + k_2 \bar{a}^2 \bar{d} + k_3 \bar{a}^2 \bar{L} + k_4 \bar{a}^2 + k_5 \bar{a} \bar{d} \bar{L} + k_6 \bar{a} \bar{d}, \\ N_2 = k_7 \bar{a} \bar{L} + k_8 \bar{d} \bar{L} + k_9 \bar{a} + k_{10} \bar{d} + k_{11} \bar{L} + k_{12}, \\ D_1 = k_{13} \bar{a} \bar{d} \bar{L} + k_{14} \bar{a} \bar{d} + k_{15} \bar{a} \bar{L} + k_{16} \bar{d} \bar{L} + k_{17} \bar{a} + k_{18} \bar{d} + k_{19} \bar{L} + 1. \end{cases} \quad (25)$$

The procedure of fitting “optimal” coefficients leads to a set of values which are presented in Table 2. The maximum relative error of formula (24) with the coefficients from Table 2 is 4.2 %, the average relative error is 1.1 %.

Table 2

The «optimal» values of the coefficients for formula (24)

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}	k_{17}	k_{18}	k_{19}
-0.084	6.21	0.24	-12.95	$-2 \cdot 10^{-3}$	-2.06	-0.07	-0.01	15.56	0.15	0.013	1.1	-0.18	-12.31	1.75	-0.013	12.52	-0.99	0.027

The “simplified” formula,

$$\frac{R_r}{h_0} = \bar{a}(1 - \bar{d}) \frac{0.233 \bar{a}^2 \bar{d} - 0.68 \bar{a}^2 + 1.066 \bar{a} + 0.105}{(1 - \bar{d}) \cdot (\bar{a} + 0.094) + \bar{L} (0.133 \bar{a} + 0.00115)}. \quad (26)$$

gives the maximum relative error of 5.1 % and average relative error of 1.3 %.

4.3. Formulas for estimating the horizontal component of the solitary wave impact

To construct the “complete” formula for the horizontal component of the solitary wave force on a partially immersed, fixed body, we assume that: The specific dependences on the body submergence d and its length L are well approximated by fractional linear functions, and the dependence on the initial wave amplitude a_0 is well approximated by fractionally rational function, in which the numerator degree is 3 and the denominator degree is 1; this component must be zero when $a_0 = 0$; this component must be independent of L when $\bar{d} = 1$.

Then the “complete” formula can be presented as

$$\frac{F_h}{\rho g h_0^2} = \bar{a} \frac{\bar{L} (k_1 \bar{a}^2 \bar{d} + k_2 \bar{a}^2 + \bar{a} \bar{d} + k_3 \bar{a} + k_4 \bar{d} + k_5) + (1 - \bar{d}) \cdot (k_6 \bar{a}^2 + k_7 \bar{a} + k_8)}{\bar{L} (k_9 \bar{a} \bar{d} + k_{10} \bar{a} + k_{11} \bar{d} + k_{12}) + (1 - \bar{d}) \cdot (k_{13} \bar{a} + k_{14})}. \quad (27)$$

The resulting “optimal” values of the coefficients are given in Table 3.

Table 3

The «optimal» values of the coefficients for formula (27)

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}
−0.8	0.77	0.039	0.016	$-2.9 \cdot 10^{-4}$	0.41	−0.075	$2.2 \cdot 10^{-4}$	$-3.7 \cdot 10^{-3}$	0.48	$1.7 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	0.847	0.32

Formula (27), with coefficients listed in Table 3, agrees well with the calculated data, yielding a maximum relative error of 6.2 % and an average relative error of 1.2 %. Applying the previously described simplification procedure to this formula results in:

$$\frac{F_h}{\rho g h_0^2} = \bar{a} \frac{\bar{L} (0.785(1-\bar{d})\bar{a}^2 + \bar{a}\bar{d} + 0.0329\bar{a} + 0.0132\bar{d}) + (1-\bar{d}) \cdot (0.741\bar{a}^2 - 0.109\bar{a})}{\bar{L} (0.477\bar{a} + 0.00694) + (1-\bar{d}) \cdot (0.986\bar{a} + 0.286)}. \quad (28)$$

The maximum relative error of this formula is 6.8 %, with an average relative error of 1.3 %.

From (28) we can estimate the maximum impact of the solitary wave on a vertical wall ($\bar{d} = 1$):

$$\frac{F_h}{\rho g h_0^2} = \bar{a} \frac{1.0329\bar{a} + 0.0132}{0.477\bar{a} + 0.00694}. \quad (29)$$

Liu and Al-Banaa [14] derived a formula for this case based on their laboratory experiments and numerical calculations using the Navier–Stokes model:

$$\frac{F_h}{\rho g h_0^2} = 2.65 \left(1 - \exp(-\bar{a}^{1.14}) \right). \quad (30)$$

Gusev, Khakimzyanov, and Chubarov [13] approximated the results of calculations performed within the framework of the Pot model using the formula:

$$\frac{F_h}{\rho g h_0^2} = 0.11\bar{a}^2 + 2.19\bar{a}. \quad (31)$$

Figure 6 presents the curves corresponding to formulas (29)–(31), alongside experimental data from Chen and Wang [35] and numerical results from Cooker, Weidman, and Bale [33]. The curves obtained from these formulas and the numerical calculations show good agreement over the range of input parameters considered.

4.4. Formulas for estimating the vertical component of the solitary wave impact

To construct the “complete” formula for the vertical component of the impact force on a partially immersed, fixed body, we assume that: in slices, the dependence on d is very weak and is well described by a linear function; the dependence on L is also well described by a linear function; the dependence on the initial wave amplitude a_0 is well described by a cubic function; the vertical component of the force is zero when $a_0 = 0$ and is also zero

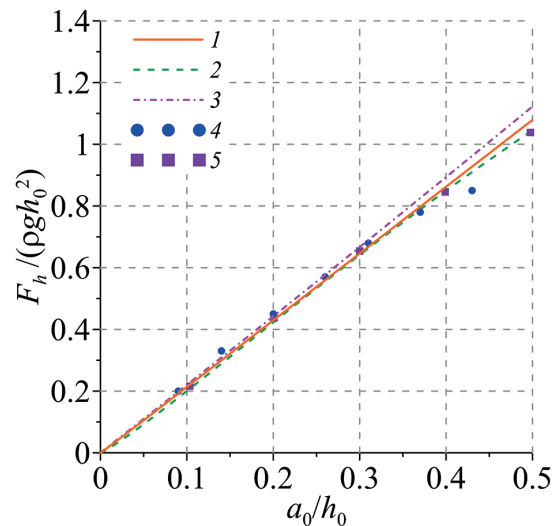


Fig. 6. Dependences of the maximum wave impact on the vertical wall on the amplitude of the solitary wave a_0 , determined by formulas (29) — 1, (30) — 2, (31) — 3; results of the laboratory experiments by [35] — 4; results of the calculations by [33] — 5

when $L = 0$; when $\bar{d} = 1$, the vertical component of the force is formally zero; however, these cases were excluded from the formula construction because when $\bar{d} \rightarrow 1$ the defined vertical component does not approach zero.

Then the “complete” formula can be presented as:

$$\frac{F_v}{\rho g h_0^2} = \bar{L} \cdot \left(\bar{a}^3 (k_1 \bar{d} + k_2) + \bar{a}^2 (k_3 \bar{d} + k_4) + \bar{a} (k_5 \bar{d} + k_6) \right). \quad (32)$$

The “optimal” values of the coefficients are listed in Table 4. The maximum relative error of formula (32) with the coefficients from Table 4 is 4.3 %, the average relative error is 0.7 %.

Table 4

The «optimal» values of the coefficients for formula (32)

k_1	k_2	k_3	k_4	k_5	k_6
0.83	-0.06	-0.48	-0.5	0.013	1.01

By iteratively setting the least significant coefficients to zero and re-optimizing at each step, a simplified formula for estimating the vertical component of the force was obtained:

$$\frac{F_v}{\rho g h_0^2} = \bar{L} \left(0.351 \bar{a}^3 \bar{d} - \bar{a}^2 (0.265 \bar{d} + 0.492) + \bar{a} \right).$$

The maximum relative error of the formula is 4.1 %, with an average relative error of 1 %.

Thus, we propose the analytical relations (19), (26), (28), and (33) for estimating the maximum runups on the front and back sides of a partially immersed, fixed body, as well as the horizontal and vertical components of the solitary-wave force. These formulas are recommended for use with the corresponding maximum relative errors of 3.7 %, 5.1 %, 6.8 %, and 4.1 %, respectively.

4.5. Example of the formulas application

In this subsection, we compare the results of the proposed formulas with the calculations of Ma et al. [10], obtained using OpenFOAM and a nonhydrostatic model based on the Euler equations. These authors considered a solitary wave with amplitude $a_0 = 0.1$ m propagating in a basin of depth $h_0 = 1$ m, interacting with a partially immersed body of submergence $d = 0.4$ m and length $L = 5$ m.

The value of the horizontal component of the force was determined by OpenFOAM algorithms to be 554 N, while the non-hydrostatic model gave 580 N, and formula (28) yielded 514 N. The values of the vertical component of the force calculated by the OpenFOAM algorithms and within the non-hydrostatic model are almost identical in magnitude ($F_v = 4501$ H), while formula (33) gives the value $F_v = 4614$ H. Accordingly, the relative error of the proposed formula is 7 % for the horizontal component compared with OpenFOAM and 11 % compared with the non-hydrostatic model, while the relative error for the vertical component is less than 3 %.

It should be noted that these relative errors largely reflect differences between the SGN model used in this study and the models employed by Ma et al. [10]. Calculations using the SGN model give values of $F_h = 510$ H and $F_v = 4670$ H, which is very close to the results of the formulas.

5. Conclusion

In this study, we present the construction of analytical relations for estimating the characteristics of the impact of a vertical bore and a solitary wave on a partially immersed, fixed body. These relations are obtained by approximating the results of a series of calculations in which key problem parameters — namely, the body submergence, body length, and incoming wave amplitude — were systematically varied. The bore and solitary-wave impacts were simulated using one-dimensional shallow water models of the first and second long-wave approximations, respectively, employing numerical algorithms previously developed and validated by the authors. These algorithms are one to two orders of magnitude faster than those based on the Pot model, enabling the computation of several thousand scenarios.

To estimate the bore impact, 360 calculations were performed, varying the body submergence from $0.05h_0$ to h_0 and bore amplitude from $0.01h_0$ to $0.5h_0$. Formulas (12), (14) and (16) are constructed to estimate the maximum runup, and horizontal and vertical components of the total wave force, respectively, with a maximum relative error of only 0.7 %.

To obtain the estimates for the solitary wave impact, 3055 calculations were performed, varying the body submergence from $0.05h_0$ to h_0 , the body length from $3h_0$ to $25h_0$, and the wave amplitude from $0.01h_0$ to $0.5h_0$. Formulas (19), (26), (28) and (33) are constructed to estimate the maximum runup on the front and back sides of the body, as well as the horizontal and vertical components of the total wave force, with a maximum relative error of 6.8 %.

It should be noted that the nonlinear dispersive SGN model can underestimate the front-side runup and the horizontal component of the wave force by up to 10 % within the considered parameter ranges. Therefore, for conservative upper estimates, it is recommended to multiply the values predicted by formulas (19) and (28) by 1.1.

We compared the results obtained using the constructed formulas with solutions from other studies. The analysis indicates that these formulas can be applied reliably within the considered ranges of the problem parameters.

It should be noted that the presented formulas were derived for the most simplified formulation of the problem. This formulation does not account for non-uniformity of the interaction along the second horizontal direction, irregularities in the basin floor and the body base, the actual shape of the structure, the angle of wave incidence, or variations in wave form. Consequently, the formulas are suitable only for preliminary, rough estimates of wave impact characteristics. More detailed physical or numerical modeling is required to incorporate these factors for a specific problem. Nevertheless, the derived formulas, even in this simplified form, can inform updates to existing regulatory guidelines (e. g., [36]).

Future research may focus on extending the applicability of these formulas by employing more accurate models, accounting for waves reflected from the shore, and considering other wave types — such as single waves with variable length or N-waves.

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Conflict of interests

The authors declare no conflict of interests

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