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© *I. A. Sergievskaya*[1,2]*, *S. A. Ermakov*[1,2], *T. N. Lazareva*[1], 2025

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[1]A. V. Gaponov-Grekhov Institute of Applied Physics RAS, 46 Ulyanova Str., Nizhny Novgorod 903950, Russia
[2]Volga State University of Water Transport, 5 Nesterova Str. Nizhny Novgorod 903950, Russia
*i.sergia@ipfran.ru

MODE TRANSFORMATION OF WAVES ON THE SURFACE OF A LIQUID COVERED BY AN ELASTIC FILM OF FINITE THICKNESS

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Abstract

The study of surface wave suppression due to oil product films and biogenic films in areas of catastrophic phytoplankton blooms is an important task in application to the problem of remote diagnostics of pollution on the sea surface. The peculiarity of such films in comparison with the well-studied case of quasi-monomolecular films of surfactants is a significant (on the order of or more than 1 micron) film thickness, the latter in this case is described as a layer of viscous liquid. This paper investigates wave damping on a water surface covered by a layer of another viscous fluid of finite thickness with an elastic boundary between them within the framework of linear theory. The features of two different types of wave modes, which for infinitely thin film are characterized as transverse (gravitational-capillary waves, GCW) and longitudinal (Marangoni waves, MW), are numerically analyzed. The evolution of these modes with increasing thickness of the top layer up to thicknesses much larger than the thickness of the viscous sublayer in the film is analyzed. It is shown that in some interval of interface elasticity, determined by the wavelength and viscosity of the top layer, a mutual transformation of the modes occurs at the thickness of the layer of the order of viscous sublayer thickness in the film. Namely, a wave that was GCW for an infinitely thin film, at film thicknesses greater than the thickness of the viscous sublayer, transitions to a MW, and vice versa. This effect arises because the GCW and MW are neither purely gravity-capillary nor purely dilatational. Laboratory experiments showed good agreement with the numerical results and confirmed the existence of the mode transformation effect.

Keywords: gravity-capillary waves, Marangoni waves, two-layer fluid, elastic film

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[1]Институт прикладной физики им. А. В. Гапонова-Грехова РАН, 903950, Нижний Новгород, ул. Ульянова, д. 46
[2]Волжский государственный университет водного транспорта, 903950, Нижний Новгород, ул. Нестерова, д. 5
*i.sergia@ipfran.ru

МОДОВАЯ ТРАНСФОРМАЦИЯ ВОЛН НА ПОВЕРХНОСТИ ЖИДКОСТИ, ПОКРЫТОЙ УПРУГОЙ ПЛЕНКОЙ КОНЕЧНОЙ ТОЛЩИНЫ

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Аннотация

Исследование подавления поверхностных волн пленками нефтепродуктов и биогенными пленками в областях катастрофического цветения фитопланктона является актуальной задачей в приложении к проблеме дистанционной диагностики загрязнений на морской поверхности. Особенностью таких пленок по сравнению с хорошо изученным случаем квазимономолекулярных пленок поверхностно-активных веществ является значительная (порядка и более 1 мкм) толщина пленки, последнюю в этом случае описывают как слой вязкой

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жидкости. В работе в рамках линейной теории исследовано затухание волн на поверхности воды, покрытой слоем другой вязкой жидкости конечной толщины с упругой границей между ними. Численно проанализированы особенности двух разных типов волновых мод, которые в пределе бесконечно тонкой пленки характеризуются как поперечные (гравитационно-капиллярные волны, ГКВ) и продольные (волны Марангони, ВМ). Проанализирована эволюция этих мод с ростом толщины верхнего слоя вплоть до толщин, много больших толщины вязкого подслоя, в пленке. Показано, что в некотором интервале упругостей границы раздела, определяемым длиной волны и вязкостью жидкостей, при толщине верхнего слоя порядка толщины вязкого подслоя в пленке, происходит взаимная трансформация мод. Именно волна, которая была ГКВ для бесконечно тонкой пленки, при толщинах пленки, превышающих толщину вязкого подслоя, переходит в ВМ, и наоборот. Этот эффект возникает из-за того, что ГКВ и ВМ не являются ни чисто гравитационно-капиллярными, ни чисто дилатационными. Лабораторные эксперименты показали хорошее согласие с результатами численного анализа и подтвердили существование эффекта модовой трансформации.

Ключевые слова: гравитационно-капиллярные волны, волны Марангони, двухслойная жидкость, упругая пленка

1. Introduction

The suppression of surface waves in the presence of films of various origins, including oil product films and biogenic films, is a relevant issue in the context of remote sensing of pollution on the ocean and inland water surfaces (see, e. g., [1, 2]). A distinguishing feature of oil films, in contrast to monomolecular films of pure surfactants, is their significant thickness, which typically ranges from 0.1 to 10 μm [3] and can be orders of magnitude greater in the case of major spills. Notably, biogenic films formed in regions of intense phytoplankton blooms can also exhibit substantial thickness (see [4]). Such *thick* films should be treated as layers of viscous fluid with elastic boundaries, and wave damping at the interface must be analyzed within the framework of a two-layer fluid model, where the system consists of *water — thick surface film*.

The damping of waves on a liquid surface covered by a monomolecular film has been extensively studied, with two types of surface oscillations described in detail: gravity-capillary waves (GCW) and Marangoni waves (MW). Expressions for the dispersion equation and the damping ratio of GCW have been derived in the general case using the linearized Navier-Stokes equation with boundary conditions that account for the presence of an elastic film on the water surface (see, e. g., [5–11]). The observed maximum in the GCW damping ratio corresponds to the case where the wavenumbers of MW and GCW are close at the same frequency. Lucassen [9] attributed this maximum to resonance between GCW and MW but noted that this does not imply direct interaction between these two wave types. However, several studies (see, e. g., [1,10], as well as references cited in [14]) suggest that MW extract energy from GCW, with this process being most efficient when the frequencies and wavenumbers of these two modes are similar, leading to the observed damping maximum. In [12–14], the investigation of wave motion was based on an initial decomposition of velocity into vorticity-dominated and potential components. The assumption of purely vortical motion allowed for the demonstration that the horizontal velocity component of MW is significantly greater than the vertical component, implying that MW can be considered quasi-horizontal, confined to the surface, and attenuating over a distance on the order of the wavelength. It was shown that the vortical component of GCW can be mathematically described as a "forced" Marangoni mode excited by the potential component. In [15–17], wave dynamics on a liquid surface covered by an elastic film with complex elasticity (nonzero shear viscosity) and surface tension were examined, revealing that mutual transformation between MW and GCW modes is possible. The authors of these studies noted that while the physical interpretation of the complex nature of these parameters remains unclear, the potential coupling between MW and GCW cannot be disregarded.

The case in which the surface film has a finite thickness presents a more complex problem. In [18], a system consisting of two viscous layers of different fluids with elastic films at the upper and lower boundaries of the top layer was considered. Wave motion is described by the linearized Navier-Stokes equation with boundary conditions that account for the elasticity of the interfaces. This system is assumed to represent, for example, an oil film on the water surface. The formation of a film at the upper boundary of the oil layer can be attributed to the complex nature of oil, which consists of various fractions that contribute to film formation. Within this model, a general solution for GCW was obtained for the case in which the film thickness is much smaller than the thickness of the viscous boundary layer [18]. Based on the decomposition of waves into vortical and potential components, analytical expressions for the dispersion equation and the damping ratio of GCW were derived for both a thin film and a thick top layer [19]. In the latter case, when an elastic film is present at both boundaries of the top layer, the existence of two MW modes localized at the lower and upper interfaces was demonstrated, leading to two corresponding maxima in the damping ratio.

The derivation of an analytical solution for films of intermediate thickness is not feasible. In this study, a numerical investigation of wave damping on the surface of a two-layer fluid is conducted for a wavelength of 2 cm. The choice of wavelength is motivated by the fact that microwave radars, commonly used for the remote sensing of wind-induced surface waves, operate at Bragg wavenumbers of a few centimeters when observing at moderate incidence angles (e. g., in satellite-based radar systems). A numerical analysis is performed to describe the evolution of two wave modes, which correspond to GCW and MW in the case of an infinitely thin film, as the thickness of the top layer increases. It is shown that within a certain range of film elasticities, determined by the wavelength and the viscosity of the fluids, mode transformation occurs when the thickness of the top layer approaches that of the viscous sublayer in the film. Laboratory experiments demonstrate good agreement with the numerical results and confirm the existence of this transformation effect.

2. Description of the Model and Numerical Methodology

The analysis of waves on the surface of a medium consisting of two layers of different viscous Newtonian fluids is conducted in a two-dimensional formulation, where z — represents the vertical axis and wave propagation occurs along the horizontal axis x. An interfacial film with elasticity E is located at the boundary between the two fluids ($z = 0$). The thickness of the upper fluid layer is denoted as h, while the lower layer is assumed to have infinite depth for simplicity. The vertical (W) and the vertical and horizontal (U) velocity components and P pressure at each point within the medium, are governed by the Navier-Stokes equations and the continuity equation:

$$\begin{gathered} U_t + p_x / \rho = \nu \nabla^2 U , \\ W_t + p_z / \rho + gz = \nu \nabla^2 W , \\ U_x + W_z = 0 . \end{gathered} \tag{1}$$

Here ρ and ν denote the density and kinematic viscosity of the medium, respectively. Subscripts x and z indicate differentiation with respect to the corresponding coordinates. The solution of system (1) takes the form $W_{f,\,w}(z) \cdot \exp(-i\omega t + ikx)$, where is the oscillation frequency, $\mathrm{Im}\omega$ is the wave damping ratio, and k is the wavenumber. Subscripts refer to the upper (f) or lower (w) layers. in the top layer $W_f(z) \propto c_1\exp(-kz) + c_2\exp(kz) + c_3\exp(-l_f \cdot kz) + c_4\exp(l_f \cdot kz)$, in the lower layer $W_w(\mathrm{z}) \propto b_1\exp(-kz) + b_3\exp(-l_w \cdot kz)$, where $l_{f,w} = \sqrt{\dfrac{\omega}{\nu_{f,w}} + 1}$, $d_{f,w} = \dfrac{\sqrt{2}}{l_{f,w}}$ represent the thicknesses of the viscous boundary layers in the respective fluids. Terms containing $\exp(\pm kz)$, correspond to the potential component, while terms with $\exp(\pm l_{f,\,w} \cdot kz)$ represent the vortical component, with $c_{1,2,3,4}$, $b_{1,2}$ denoting the amplitudes of the respective harmonics.

To determine the amplitudes, kinematic and dynamic boundary conditions are applied. The dynamic boundary conditions, both normal and tangential, take the following form (see, for example, [18]) for the upper boundary at $z = h$:

$$\begin{gathered} P_f - gz - 2\nu_f W_{fz} + (\sigma_{af} / \rho_f) Z_{xx} = 0, \\ \rho_f \nu_f (U_{fz} + W_{fx}) = 0 , \end{gathered} \tag{2}$$

and the lower boundary at $z = 0$

$$\begin{gathered} \rho_f \left(P_f - \mathrm{g}z - 2\nu_f W_{fz} \right) = \rho_w \left(p_w - \mathrm{g}z - 2\nu_w W_{w_z} \right) + \sigma_{fw} \zeta_{-xx} , \\ \rho_f \nu_f (U_{fz} + W_{fx}) + \mathrm{E}\frac{\partial^2 \xi}{\partial x^2} = \rho_w \nu_w (U_{wz} + W_{wx}) , \end{gathered} \tag{3}$$

where $\sigma_{af,fw}$ is the surface tension at the upper and lower boundaries of the top layer, g is the gravitational acceleration, Z is the vertical displacement of the surface, ξ — is the horizontal shear, respectively $W = \dfrac{\partial Z}{\partial t}$, $U = \dfrac{\partial \xi}{\partial t}$.

This system describes all waves that can be excited in a two-layer fluid. The focus here is on two types of oscillations, which, in the case of an infinitely thin (monomolecular) film on the upper surface of the lower fluid, correspond to GCW and MW. A monomolecular film in this context corresponds to $h = 0$, and the dispersion equation and damping ratio for GCW take the following form (see also [14]):

$$(\mathrm{Re}\,\omega)^2 = gk + \left(\sigma_{af} + \sigma_{fw}\right)k^3, \tag{4}$$

$$\mathrm{Im}\ \omega = 2\nu_w k^2 \frac{1 - X + X\cdot Y}{1 - 2X + X^2},\quad X = \frac{Ek^2}{\rho_w\sqrt{2\nu_w\omega^3}},\quad Y = \frac{Ek}{4\rho_w\nu_w\omega},$$

for MW [14]:

$$k^2 = \frac{i+1}{E}\sqrt{\frac{\rho_w\eta_w\omega^3}{2}}. \tag{5}$$

If the thickness of the top layer h exceeds the wavelength of the GCW, the damping ratio and dispersion equation for GCW are expressed as:

$$\mathrm{Im}\,\omega = 2\nu_f k^2,$$

$$(\mathrm{Re}\,\omega)^2 = gk + \sigma_{af}k^3. \tag{6}$$

In this case, the MW is confined to the interface between the layers, and its dispersion relation and damping ratio are determined by the expression given in [19].

$$k^2 = \frac{i+1}{E}\left(\sqrt{\frac{\rho_f\eta_f\omega^3}{2}} + \sqrt{\frac{\rho_w\eta_w\omega^3}{2}}\right). \tag{7}$$

In Fig. 1, the dependence of the GCW and MW frequencies on film elasticity is presented for two cases: (a) a monomolecular film on the water surface and (b) a top layer thickness much greater than the thickness of the viscous boundary layer within the film. The selected values of surface tension, 30 mN/m at both the upper and lower boundaries of the top layer, as well as a bulk viscosity of 0.1 cm^2/s, are typical for crude oil (with a density of 0.85 g/cm^3) and its derivatives [20–24], while the density and viscosity of the lower layer correspond to those of water. It is evident that there exists a range of film elasticities where the GCW frequencies (which are similar for a thin film and a thick layer) are lower than the MW frequencies for a thin film and higher than the MW frequencies for a thick top layer. The wave that corresponds to the MW in the case of a thin film is denoted as W2, while the one corresponding to the GCW is denoted as W1. It is then evident that within the specified range of film elasticities, the frequencies of W1 and W2 intersect at certain top-layer thicknesses.

In numerical calculations of the dependence of the damping ratio and wave frequency on film thickness for a given wavenumber, equations (4) and (5) at $h = 0$ are used as initial values for solving the problem at a small nonzero thickness $\delta h << d_{f,w}$. The solution obtained for $h = \delta h$ is then used as the initial condition for determining the damping ratio and frequency at $h = 2\delta h$, and so forth. This approach allows tracking the evolution of solutions for W1 and W2 as the thickness of the top layer increases. To verify the results, calculations were also performed in reverse order, from larger to smaller top layer thicknesses, using equations (6) and (7) as initial conditions.

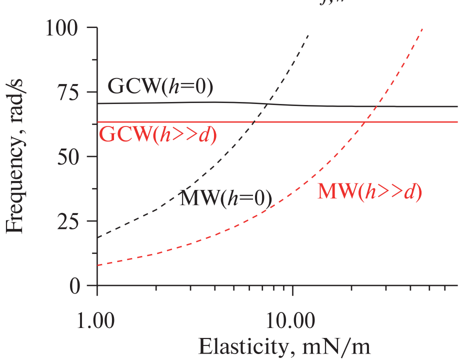

Fig. 1. Dependence of the 2 cm wave frequency on the film elasticity, solid curves are GCW, dashed curves are MW. Black curves correspond to monomolecular film, dashed — thick top layer

3. Results of Numerical Calculations

In Fig. 2, the dependencies of the damping ratio (normalized to the damping coefficient in the absence of a film $2\nu_w k^2$) and the frequencies of waves W1 and W2 on the thickness of the top layer are presented for various film elasticities at the interface. In Figs. 2a and 2b, it is evident that when the elasticity is either low (E < 15 mN/m) or high (E > 25 mN/m), the damping ratio and frequency of wave W1 correspond to GCW for both a thin film and a thick top layer, with a maximum observed in the damping ratio at certain intermediate thicknesses. However, within a certain range of elasticities (15 < E < 20 mN/m), wave W1, which corresponds to GCW for a thin film, transitions to MW at larger thicknesses, indicating a mode transformation. Wave W2 (Figs. 2c and 2d) behaves as MW for both thin films and thick top layers when E < 15 mN/m and E > 25 mN/m, while in the range 15 < E < 25 mN/m, it transitions from MW to GCW. It is noteworthy that the elasticity range in which this transition occurs is slightly smaller than the range where GCW frequencies are lower than MW frequencies for a thin film and higher than MW frequencies for a thick top layer (Fig. 1). The transformation between these oscillation modes occurs abruptly at a bifurcation point determined by specific values of elasticity and top layer thickness, which is on the order of the boundary layer thickness.

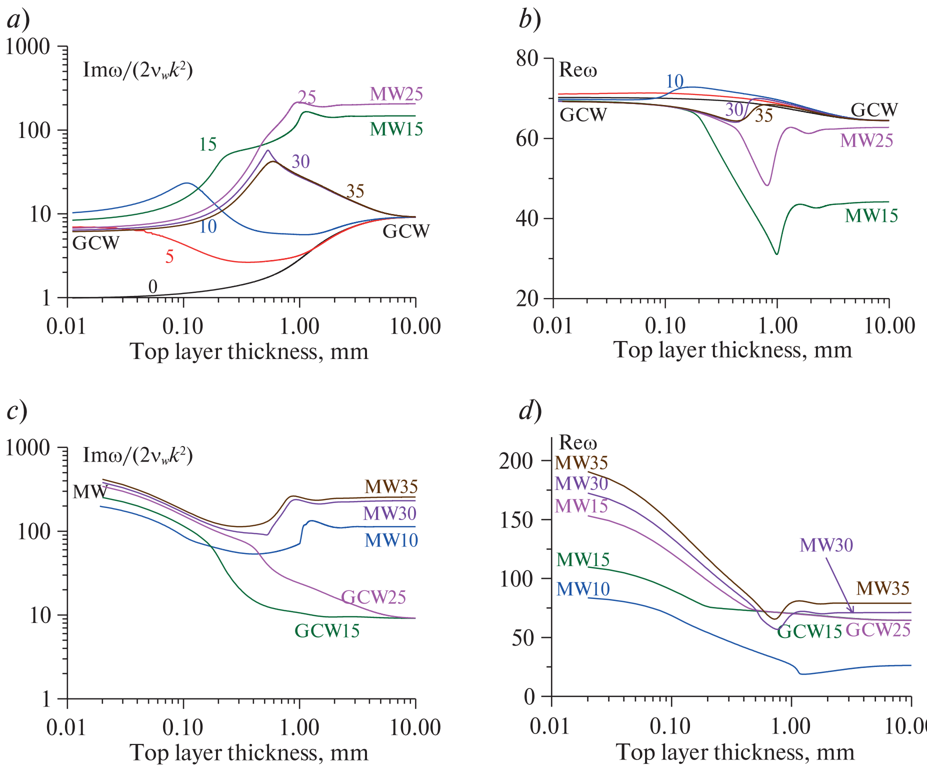

Fig. 2. Dependence of the damping ratio (a, c) and frequency (b, d) on the thickness of the top layer for waves W1 (a, b) and W2 (c, d). The numbers near the curves indicate elasticity in mN/m, wavelength = 2 cm

4. Discussion of Numerical Calculation Results

A detailed analysis is conducted to examine the transformation of MW and GCW. In Fig. 3, the dependencies of the wave damping ratio and wave frequency on film thickness near the intersection point of damping ratio W1 and W2 are presented. It is evident that wave frequencies intersect at lower elasticities and smaller thicknesses than the damping ratio. The magnitude of the local maximum of the damping ratio W1 increases with elasticity, while the minimum of W2 decreases until they become equal. In this case, this corresponds to an elasticity of approximately 12 mN/m. At higher elasticity values, the behavior of W1 and W2 changes significantly: W1 increases, while W2 decreases (i. e., W1 and W2 intersect). At this bifurcation point, a change in frequency behavior is also observed: a maximum appears on W1, and a minimum on W2, with no further intersections at higher elasticity values. Evidently, a second bifurcation point exists at even greater elasticity values, where the damping ratio ceases to intersect, while the frequency curves begin to do so. Similar bifurcation points have been described for thin films with shear and complex viscosity in [15–17].

In Fig. 4, the wave horizontal velocity profiles for W1 and W2 are presented for two film elasticity values (25 and 60 mN/m). It is evident that at an infinitesimally thick film, the velocity profile of W1 (Fig. 4a) exhibits characteristics typical of GCW at both elasticity values. In the presence of a film, the horizontal velocities at the surface are directed opposite to the particle velocities below the boundary layer [14]. The velocity profile of W2 (Fig. 4b) is characteristic of MW, with the wave being confined to the surface. At a greater upper-layer thickness (1 cm), the velocity profile of W1 (Fig. 4c) corresponds to GCW at an elasticity of 60 mN/m, whereas at 25 mN/m, the wave is confined to the interface, which is typical for MW (see [19]). In this case, the velocity profile of W2 (Fig. 4d) at 25 mN/m demonstrates similarity to GCW, while at 60 mN/m, it corresponds to MW. Thus, the results presented in Fig. 4 are consistent with those in Fig. 2.

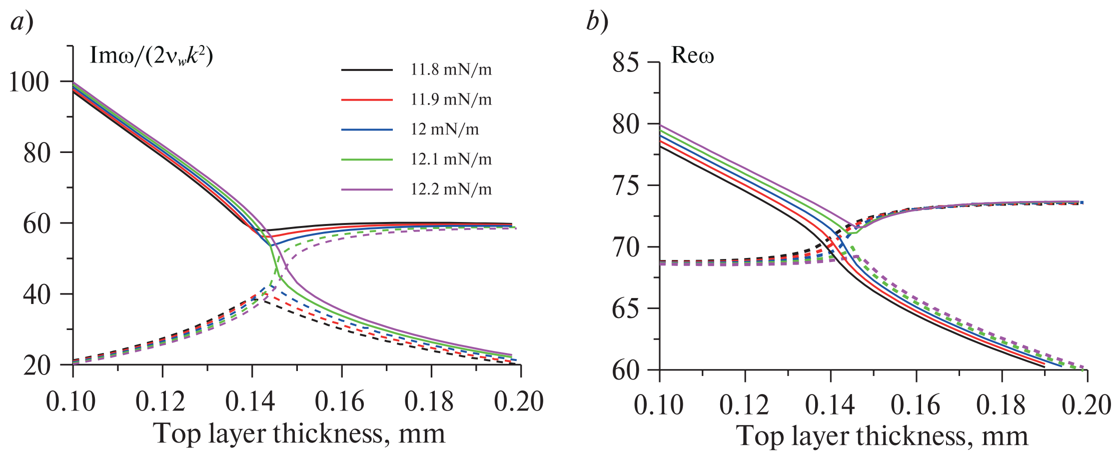

Fig. 3. Dependence of the wave damping ratio (a) and frequency (b) on the thickness of the top layer near the bifurcation point. The wavelength is 2 cm

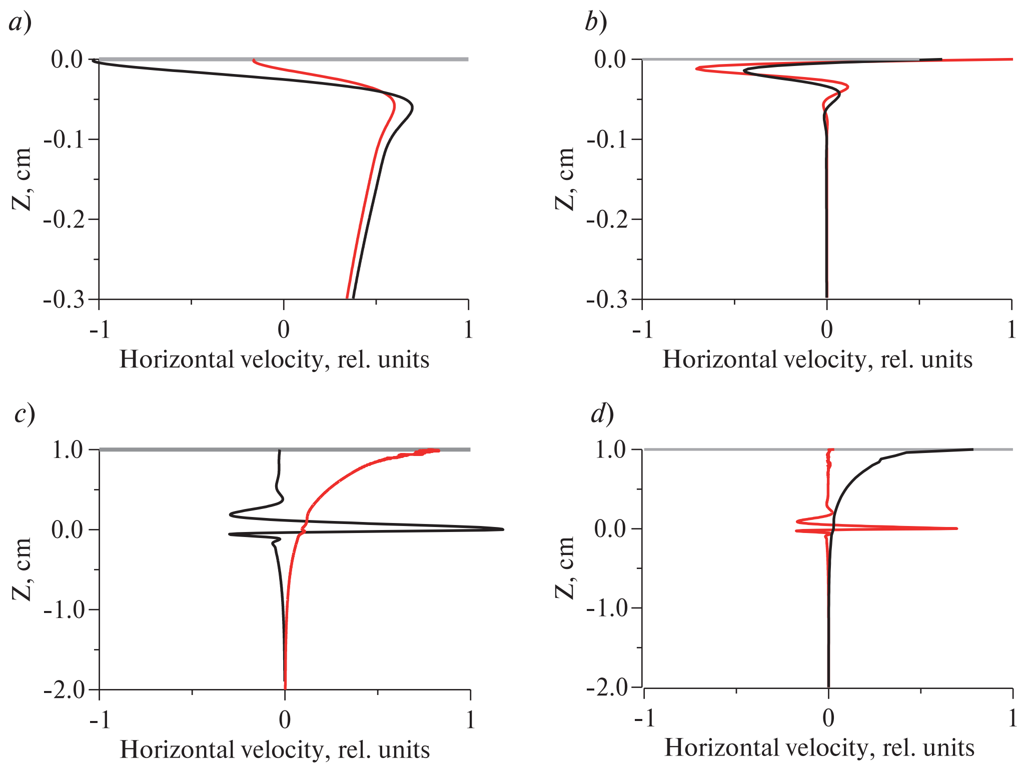

Fig. 4. Horizontal velocity profiles of waves W1 (a, c) and W2 (b, d) for a thin film (a, b) and a thick top layer (c, d). Black curves correspond to E = 25 mN/m, red curves to E = 60 mN/m. The gray straight line represents the upper boundary of the top layer

5. Laboratory Experiment

Laboratory measurements were conducted to examine the characteristics of waves on water covered by a finite-thickness layer of viscous liquid. Waves were generated in a tank filled with water and covered by an oil layer. The wave excitation occurred due to the effect of parametric resonance under vertical oscillations of a vibration stand, on which the tank was placed. The frequency of parametrically excited waves was 20 Hz, which corresponded to a GCW wavelength of approximately 2 cm in clean water. The wave damping ratio was determined based on the excitation threshold of natural standing modes, while the wavelength was measured from photographs. The methodology of the parametric method has been described in detail in [25]; therefore, only the obtained results are briefly presented here. The measured dependencies of the wave damping ratio and the effective surface tension coefficient (EST = $(\omega^2 - gk)/k^3)$) as a function of oil layer thickness are shown in Fig. 5. A portion of the data (black symbols) was taken from [23] and obtained at a water depth of 2 cm, while new experiments (red symbols) were conducted at a water depth of 10 cm. The results of different experiments are in good agreement. Fig. 5 also presents calculated curves for oil with an interfacial elasticity of 30 mN/m at the oil/water boundary, showing satisfactory agreement between calculations and experimental results.

At small film thicknesses, gravitational-capillary waves (GCW, W1) are excited due to parametric resonance, observed at the surface due to the vertical displacement of the water surface. As the film thickness increases, the dispersion relation for W1 and W2 waves changes; both waves contain both longitudinal and transverse components, which lead to vertical displacement of the surface. The maximum of the damping ratio in the experiment approximately coincides with the point where the damping ratio dependencies for W1 and W2 intersect. Since waves with the least damping are excited in the experiment, after the intersection point, W2 waves are excited first, which corresponds to a different dependency of the effective surface tension on the thickness. As a result, the transition of the EST from W1 to W2 occurs. Since the EST dependencies do not intersect at these thicknesses, the transition from W1 to W2 happens discontinuously, which confirms the existence of the mode transformation effect.

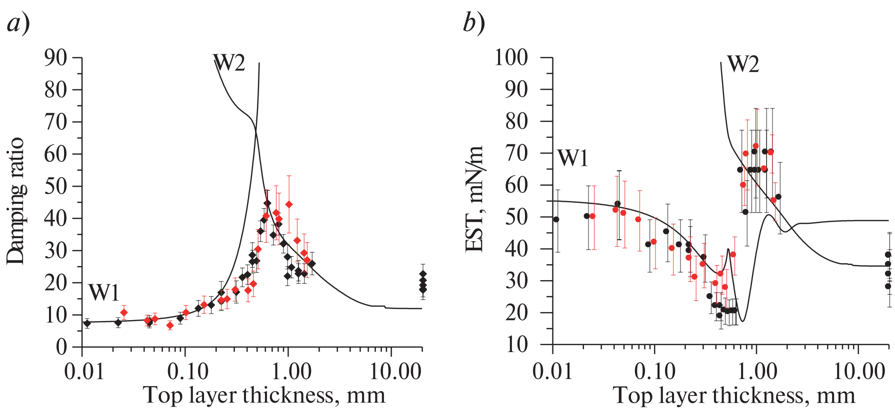

Fig. 5 Damping ratio (a) and effective surface tension coefficient (b) as functions of the oil layer thickness on the water surface. Symbols represent experimental data, and curves correspond to numerical calculations

6. Conclusion

A numerical study was conducted on wave damping at the surface of a liquid consisting of two viscous layers of finite thickness with an elastic film between them. It was demonstrated that the two types of oscillations, which are predominantly transverse (GCW) and longitudinal (MW) in the case of an infinitesimally thin top layer, do not retain their purely longitudinal or transverse nature as the top layer thickness increases. This study has shown that for certain values of film elasticity, the mode that corresponds to GCW in the limit of an infinitesimally thin film transforms into MW when the top layer is significantly thicker than the viscous boundary layer. Simultaneously, the mode that initially corresponds to MW transitions into GCW. This indicates that both types of oscillations cease to be purely gravity-capillary or purely dilatational. The laboratory results for the wave damping ratio and effective surface tension coefficient are in good agreement with the numerical calculations.

The previously obtained approximate solutions for the wave damping ratio [16,17] are valid up to top layer thicknesses on the order of the viscous boundary layer thickness, which in many cases does not correspond to the actual thickness of the contaminating film. Numerical calculations, despite their complexity in practical use for oil spill detection systems on the sea surface when solving inverse problems, can still help assess the accuracy of the approximate equations.

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About the Authors

SERGIEVSKAYA Irina A., Head of the Laboratory IAP RAS, Cand. Sc. (Phys.-Math.), ORCID: 0000-0001-8870-2047, e-mail: i.sergia@ipfran.ru

ERMAKOV Stanislav A., Head of the Department IAP RAS, Senior Researcher, Dr. Sc. (Phys.-Math.), ORCID: 0000-0002-0869-4954, e-mail: stas.ermakov@ipfran.ru

LAZAREVA Tatyana N., Leading Programmer IAP RAS, ORCID: 0000-0002-0674-4831, e-mail: lazareva@ipfran.ru