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THE DEPENDENCE OF WAVE HEIGHT PROBABILITY DISTRIBUTIONS ON PHYSICAL PARAMETERS FROM MEASUREMENTS NEAR SAKHALIN ISLAND

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Abstract

The data of long-term surface waves measurements with bottom sensors near Sakhalin Island were used to build instrumental probability distributions for exceedance of wave heights. Waves with heights exceeding the significant wave height by more than three times were recorded. Specific features of the observations conducted during the periods of open and ice-covered sea surfaces are discussed. The subsets of statistically homogeneous data are arranged through selection considering the natural physical dimensionless parameters of the task, that control the effects of finite depth and nonlinearity (namely, the wave steepness, the wave amplitude to water depth ratio, the Ursell parameter). The manifestation of these and composite parameters in theoretical probability distributions of wave heights is discussed. The effects of nonlinearity and the measurement point depth on probability distributions are estimated with a focus on abnormally high waves. In particular, it is shown that for the place of registration an increase in the wave amplitude to water depth ratio parameter leads to a decrease in the probability of abnormally high waves. This behavior is consistent with the Glukhovsky theoretical distribution. The waves characterized by relatively large dimensionless depth parameter exhibit a higher probability of substantial exceedance of the significant height and are better described by the Rayleigh distribution.

Keywords: sea surface waves, wave height probability distribution, rogue waves, nonlinear waves, Ursell parameter, finite depth effect

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ЗАВИСИМОСТЬ ВЕРОЯТНОСТНЫХ РАСПРЕДЕЛЕНИЙ ВЫСОТ ВОЛН ОТ ФИЗИЧЕСКИХ ПАРАМЕТРОВ ПО РЕЗУЛЬТАТАМ ИЗМЕРЕНИЙ У ОСТРОВА САХАЛИН

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Аннотация

Данные длительных измерений поверхностного волнения донными датчиками у о-ва Сахалин использованы для построения инструментальных распределений вероятностей превышения высот волн. Зарегистрированы волны с высотой, превышающей значительную высоту более чем в 3 раза. Обсуждаются особенности результатов наблюдений, выполненных в периоды покрытия морской поверхности льдом и открытой воды. Для создания выборок статистически однородных данных выполнена селекция с учетом естественных физических безразмерных параметров задачи, контролирующих эффекты конечной глубины и нелинейности (крутизна, отношение амплитуд волн к глубине, параметр Урселла). Обсуждается проявление этих и производных от них параметров в теоретических распределениях вероятностей высот волн. Оценены эффекты нелинейности и глубины точки измерения на вероятностные распределения с фокусом на аномально высокие волны. В частности показано, что для места регистрации рост отношения амплитуд волн к глубине приводит к уменьшению вероятности аномально высоких волн. Это поведение согласуется с теоретическим распределением Глуховского. Волны, характеризуемые сравнительно большим параметром безразмерной глубины, демонстрируют более высокую вероятность волн с существенным превышением значительной высоты и лучше описываются распределением Рэлея.

Ключевые слова: поверхностные морские волны, распределение вероятностей высот волн, аномально высокие волны, нелинейные волны, параметр Урселла, эффект конечной глубины

1. Introduction

The measurements of surface waves with bottom pressure sensors have been performed at the Special Design Bureau for Marine Research Automation of Far Eastern Branch of RAS near Sakhalin Island in the coastal zone of the Sea of Okhotsk since 2009. These data are used to assess the probability properties of waves with a focus on rare events of extreme waves (rogue waves). The measurements were performed in a series of campaigns with the installation of one or more sensors for a period of several months, including winter periods when the surface is covered with ice. Wave height H (vertical distance from the bottom of the trough to the top of the crest) is the most commonly used characteristic of sea waves. The bank of accumulated data from measurements near Sakhalin Island already contains several thousand records of abnormally high waves (rogue waves) that meet the formal criterion for exceeding the significant wave height H_s by 2 times or more, $AI = H/H_s > 2$. These results can be found in publications [1–4]. Under the assumptions of wave linearity and narrowband spectrum, the exceedance probability of wave heights is given by the Rayleigh distribution [5]:

$$P_R(H) = \exp\left[-2\left(\frac{H}{H_s}\right)^2\right], \tag{1}$$

representing the dependence on the normalized height H/H_s. In real terms, significant height is defined as the root-mean-square surface displacement σ, $H_s = 4\sigma$, or as the average of one third of the highest waves in the sample, $H_s = H_{1/3}$. Distribution (1) is usually used as the first approximation to describe sea waves. In particular, it allows to estimate the probability of abnormally high waves: $P_R(H = 2H_s) = 3.35\cdot10^{-4}$. According to the existing estimates, the actual probability of rogue waves approximately corresponds to the following value: on average, 2–3 abnormal waves per day [2]. A more accurate estimate of the probability of rare extreme events $H > 2H_s$ and the search for conditions that increase the probability of such events are topical problems of modern oceanography.

Splitting long wave records into short intervals of 10–30 min is a standard way of increasing the statistical homogeneity of the data. It is assumed that such intervals, on the one hand, correspond to the periods of approximate stationarity of conditions characterized by a constant value of H_s, and, on the other hand, are sufficient to avoid raw errors in probability estimates associated with the finiteness of the sample. Such intervals of records contain from several tens to a couple of hundreds of individual waves. Assuming the universality of wave probability distributions with respect to the significant wave height exceedance parameter H/H_s, the data for different short intervals of records can be combined to build probability distributions of sea waves. Accordingly, the result of such processing will be a single distribution $P(H/H_s)$.

When taking into account the finite width of the spectrum, the wave nonlinearity, etc., the corrections appear in theoretical probability distributions, that violate the universality of the Rayleigh distribution. Failure to take this circumstance into account leads to building a generalized probability distribution function based on heterogeneous data, which should result in wrong estimates. In particular, such an approach can mask rare anomalous distribution functions arising under certain conditions in an array of distribution functions of the same type corresponding to much more frequent conditions. In the literature, one can find both evidence of the heterogeneity of probability distributions based on different banks of field data, and a direct discussion of distribution dependence on locations, seasons, etc. [6–8], please see the overview in [9]. Obviously, in order to increase the adequacy of the probability distributions built on the field data (especially in the range of rare events) and to determine their dependence on specific wave characteristics, additional sorting of data by statistically homogeneous conditions should be applied.

The height exceedance probability distribution constructed in the work [4] based on wave measurements near Sakhalin in 2012–2015 is reproduced in Fig. 1a. In [4], it was found that the extreme values of the heights in Fig. 1a are triggered by the events of abnormally high waves observed during the ice season. The latter was easily distinguished by the typical character of the records with almost zero background waves. This season included not only the periods when the water area was covered with solid ice, but also the winter time between those. After dividing the data array into open water and ice conditions, in the first case, the data are exceptionally well described by the Glukhovsky distribution (Fig. 1b), while the second class of conditions demonstrates significant deviations from this distribution (Fig. 1c) and contains rogue waves with the greatest exceedances of the significant height. Thus, the overall probability distribution in Fig. 1a contains at least two classes of events characterized by significantly different probabilistic properties, which cannot be concluded only on the basis of Fig. 1, *a*.

For oceanography, the standard diagrams are T_z–H_s ones reflecting the probability of wave conditions with a given significant height H_s and a given period T_z, determined for a wave sequence at zero (undisturbed) level crossing. Such a diagram for a part of the measurement data near Sakhalin Island is provided in the work [4]; Fig. 2 shows its version supplemented with new data. All recorded significant heights do not exceed half the depth of the measurement point. Data with very weak waves $4\sigma < 0.2$ were not processed. It can be assumed that the time sequences corresponding to the same or close cells on this diagram correspond mainly to similar

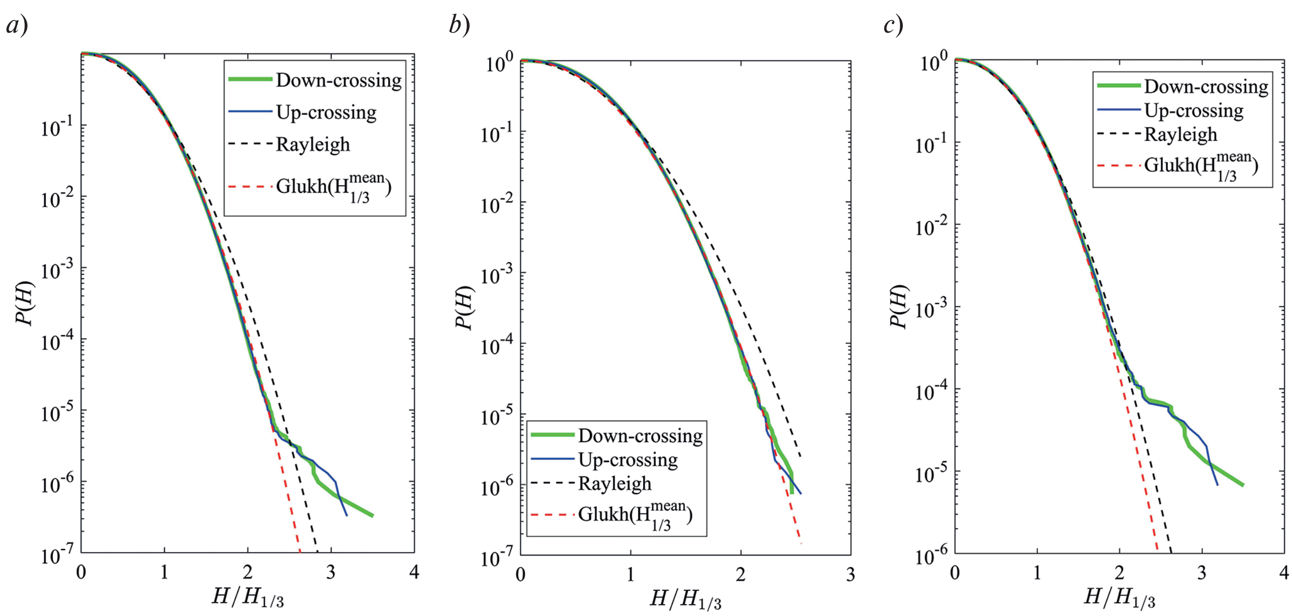


Fig. 1. Distribution of wave height exceedance probabilities according to the field data for 2012–2014 seasons: all data (*a*), ice-free period data only (*b*), and 2014–2015 season ice period data (*c*). The distributions for down-crossing and up-crossing wave heights are shown in different colors. Other lines correspond to the Rayleigh and Glukhovsky distributions, see [4]

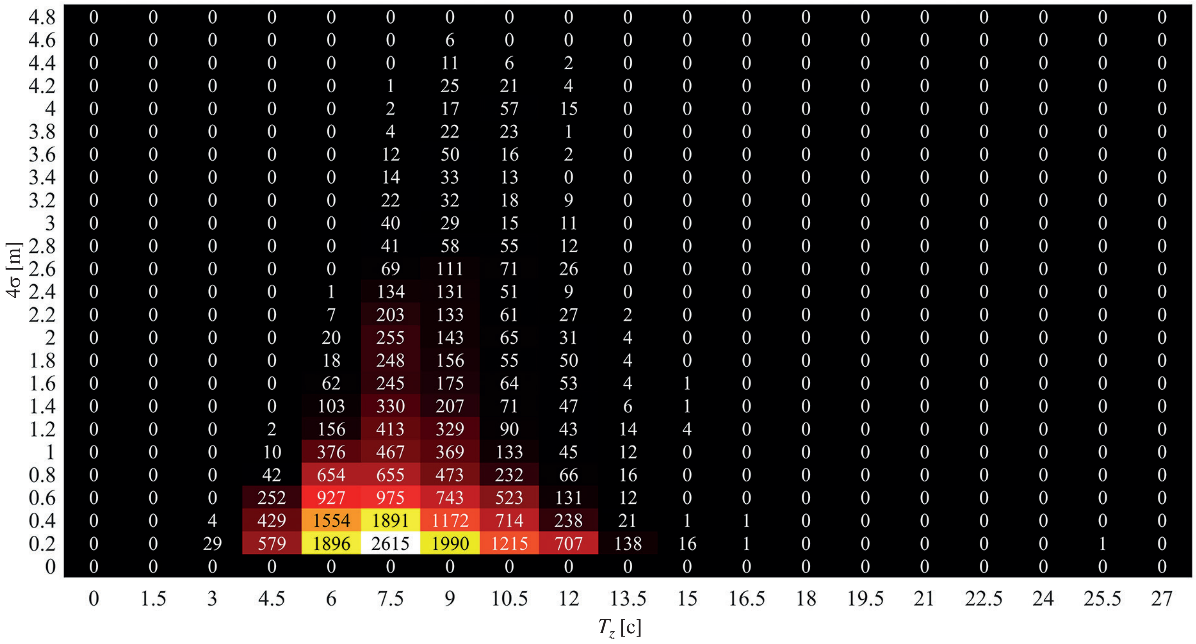


Fig. 2. The diagram for distribution of the 20-min records number (see the numbers in the cells) over significant heights $H_s = 4\sigma$ and periods T_z

weather conditions, refer to similar wave states, and are described by the same probability distribution. On the other hand, splitting the T_z–H_s table into a large number of cells leads to a significant depletion of statistical samples assumed to be homogeneous. Therefore, the selection of statistically homogeneous data subsets is an important and sophisticated task.

In this work, we propose an approach to the formation of statistical samples, based on the fulfillment of similar physical conditions expressed not in dimensional terms (as in the example of T_z–H_s diagram in Fig. 2), but in dimensionless parameters that have a clear physical meaning. It is applied to field data off the coast of Sakhalin Island. Section 2 briefly discusses intrinsic dimensionless physical parameters, which are the first candidates for the role of control parameters of probability distributions. It also discusses two possible modifications of the Rayleigh distribution, taking into account the nonlinearity. Section 3 presents the results of building probability distributions of wave heights based on field data for subsets corresponding to different intervals of dimensionless parameters. The main conclusions on the work are given in the final Section 4.

2. Physical parameters leading to a change in the probability properties of wave heights

From general considerations, the conditions at the measurement point are characterized by the local depth h, and the waves are characterized by specific length (or period) and intensity. It is well known that the properties of waves on the surface of water, associated with the finiteness of the depth h, are determined by the combination kh, where k is the wavenumber. In particular, the dimensionless depth kh is included in the dispersion relation:

$$\omega = \sqrt{gk \tanh(kh)}, \quad (2)$$

where ω is the cyclic frequency of the waves, $g = 9.81$ m/s^2 is the acceleration of gravity. The same combination determines the nonlinear properties of waves at a given depth, including the coefficients of nonlinear interactions and parametric ranges of nonlinear wave instability [10]. So, for $kh > 0.5$, homogeneous waves become modulationally unstable with respect to long modulations at an angle to the direction of propagation, and for $kh > 1.363$, longitudinal disturbances become unstable. With an increase of kh the instability increment and the range of unstable wavenumbers increase too. Since it was shown theoretically and experimentally (for example, [11]) that the conditions of modulationally stable and unstable wave systems are characterized by different probability distributions of rare events, it seems important to take into account the parameter kh when building statistical ensembles. This parameter can also be used as a characteristic of the length (period) of waves at a finite depth, therefore, instead of the period T_z, we will use the value kh (which can be understood as a dimensionless characteristic of the wave length) to describe the wave conditions.

The dimensionless parameter of wave nonlinearity in shallow water is introduced as the ratio of wave amplitude to depth. For a significant wave amplitude $H_s/2$, it can be written as $a = H_s/h/2$ [12]. In deep water, the natural size h becomes inapplicable. It is well known that in deep water the dimensionless characteristic of wave intensity is steepness, which can be written as $\varepsilon = kH_s/4 = k\sigma$. Then the nonlinearity parameter in shallow water can be represented as the ratio of steepness and dimensionless depth: $a = (kH_s)/(kh)/2 = 2\varepsilon/(kh)$. In particular, the conditions for wave breaking in shallow and deep waters are formulated in terms of the parameter threshold values, which can be written as $A/h \approx 0.25$ and $kA \approx 0.4$, respectively, where A is the wave amplitude.

In the work [13], the nonlinearity parameter μ for an arbitrary depth was discussed in the following form:

$$\mu = k\frac{H_s}{2}F(kh), \quad F = \frac{(4\tanh kh + \tanh 2kh)(1-\tanh^2 kh)}{2\tanh kh(2\tanh kh - \tanh 2kh)} + \tanh kh. \tag{3}$$

As can be seen, it also depends on depth through kh combination. In deep water this parameter becomes doubled steepness $\mu \to 2\varepsilon$, and in shallow water it is proportional to the Ursell parameter, $\mu \to 2Ur$, $Ur = 3/8\ kH_s\,(kh)^{-3}$, which, in turn, is composed of the steepness and the dimensionless depth. The Ursell parameter controls the balance between the effects of nonlinearity and dispersion that influence the wave [5]. So, for small values of Ur, the waves at shallow depths are sinusoidal, while for Ur on the order of unity, the waves become cnoidal and look like solitary humps (solitons). In the work [14], it was concluded based on direct numerical simulation of the weakly nonlinear Korteweg–de Vries equation that the probability of abnormally high waves can increase in the case of large values of Ur parameter, when the fraction of soliton-like waves was high.

Thus, the measure of wave nonlinearity can be reflected by one of the above parameters, which are interconnected through the depth parameter kh. For further research, this paper will use three of them in the following definitions:

$$\varepsilon = k\sigma, \quad a = 2\frac{\sigma}{h}, \quad Ur = \frac{3}{2}\frac{k\sigma}{(kh)^3}. \tag{4}$$

Our work [4] shows that most of the data on the results of measurements near Sakhalin Island in 2012–2015 is very well described by the Glukhovsky distribution (see Fig. 1b). Glukhovsky theoretical distribution [5]

$$P_G(H) = \exp\left[-\frac{\pi}{4\left(1+\frac{n}{\sqrt{2\pi}}\right)}\left(\frac{H}{\overline{H}}\right)^{\frac{2}{1-n}}\right], \quad n = \frac{\overline{H}}{h} \tag{5}$$

uses the nonlinearity parameter $n = \frac{\overline{H}}{h}$, introduced as the ratio of the average wave height $\overline{H} = \langle H \rangle$ to depth. It is obvious that the distribution parameter n tends to zero within the large depth (with finite H_s and increasing h) or within small amplitude waves $a \ll 1$. Then the Glukhovsky distribution (5) tends to the Rayleigh distribution (1) with the following relation: $\overline{H} = \sqrt{2\pi}H_s/4$. In the breaking zone, n approaches 0.5, and the ratio of $\overline{H}$ to H_s increases by around 20 % [5]. In general, one can roughly rewrite the normalized height using a significant height H_s: $\frac{H}{\overline{H}} \approx \frac{4}{\sqrt{2\pi}}\frac{H}{H_s}$. Then we get the relation between the parameter n and the nonlinearity parameter for the waves in shallow water: $n \approx \frac{\sqrt{2\pi}}{4}\frac{H_s}{h} = \sqrt{\frac{\pi}{2}}a$. Thus, for not too large a values, the Glukhovsky distribution (5) can be written as a function of H/H_s, which also depends on the shallow-water wave nonlinearity parameter a: $P_G(H) = P_G(H/H_s; a)$. The Glukhovsky distribution takes into account the depth effect associated with nonlinearity estimated by the parameter a. Within this distribution, in the intervals of low heights, $H < 3H_s/4$, and high heights, $H > 3H_s/4$, the probability turns out to be higher and lower than the Rayleigh distribution, respectively (see [15]).

Another example of the Rayleigh distribution modification for wave heights was discussed in [16]:

$$P_{G-Ch}(H) \approx \exp\left(-\frac{H^2}{8\sigma^2}\right)\left[1+(\lambda_4-3)B\left(\frac{H}{\sigma}\right)\right],\ \lambda_4=\frac{\langle\eta^4\rangle}{\sigma^4},\ B(\xi)=\frac{1}{384}\xi^2\left(\xi^2-16\right) \quad (6)$$

(Gram–Charlier series for a weakly non-Gaussian distribution, see [5]). Here λ_4 corresponds to the fourth statistical moment (kurtosis) for the surface displacement η. The analysis of the dependence (6) indicates that the growth of the fourth moment $\lambda_4 > 3$ leads to an increase in the probability of high waves ($H > 4\sigma$) compared to the Rayleigh distribution. The estimate of the fourth moment for deep water conditions was given in the same paper [16]:

$$\lambda_4 = 3 + 24\varepsilon^2 + \frac{\pi}{\sqrt{3}}BFI^2,\ BFI=\sqrt{2}\frac{\varepsilon}{\Delta k/k} \quad (7)$$

using the parameter of the wavenumber spectrum relative width $\Delta k/k$. The work [17] shows that the relation (7) can be significantly violated in case of unsteady waves. Other proposed estimates of the kurtosis parameter were discussed in our recent review [9]. The last component in the formula for λ_4 can be neglected in the case of a wide spectrum $\Delta k/k \gg \varepsilon$, and then the deviation λ_4 from 3 is determined by a small parameter — the square of the wave steepness. In specific cases of a relatively narrow spectrum, the modulation instability parameter *BFI* can be large, which leads to a significant increase in the high waves probability.

Thus, similarly to the Glukhovsky distribution, the distribution (6) can be written as a dependence on the normalized height and the nonlinearity parameter for deep water: $P_{G-Ch}(H) = P_{G-Ch}(H/H_s; \varepsilon)$. If consideration of nonlinearity in the distribution for shallow water (5) leads to a decrease in the high waves probability compared to the Rayleigh distribution, then for the distribution (6) the effect of nonlinearity is reversed.

3. Variability of wave height distributions according to field data depending on key physical parameters

The initial measurement data from the bottom sensors are presented as sequences of pressure records with recording frequencies of 1 Hz and 8 Hz. The task of modelling the rough surface from measurements of pressure variations at the bottom is ambiguous (in particular, waves are traditionally assumed to be unidirectional) and sophisticated. Most often, the hydrostatic theory for shallow water or the linear theory for dispersive waves is used for this purpose. Nonlinearity further complicates the problem. In this paper, the water surface displacement $\eta(t)$ was determined by the hydrostatic relation $p(t) = p_{atm} + \rho g(h_0 + \eta(t))$, where $p(t)$ is the measured pressure, p_{atm} is the atmospheric pressure, h_0 is the sensor installation depth, ρ is the water density. The hydrostatic approximation is inaccurate when the shallow water conditions $kh < 1$ are violated, so it would be better to say that the processing described here is performed for bottom pressure variations excited by surface waves in the normalized form. Due to the presence of tides, the average displacement for various 20-minute records is different, which was considered in the form of a local depth correction: $h = (\bar{p} - p_{atm})/(\rho g)$, where the pressure averaging corresponds to the given 20-minute recording interval. Atmospheric pressure was assumed to be constant and corresponding to the value at the time of the bottom station installation. The corresponding depth variation was up to 10 %.

To calculate the wave characteristics (root-mean-square surface displacement σ, heights H, periods T), oscillations with 10 min+ periods were subtracted from the records $\eta(t)$ by means of spectral filtering. Each 20-minute sequence was characterized by its own values of significant height $H_{1/3}$, root-mean-square surface displacement σ, and average wave period T_z. Assuming the dispersion relation (2) is satisfied, the wavenumber k corresponding to the frequency $\omega = 2\pi/T_z$ and the effective depth h was determined for each interval.

In this work, we used measurement data for 2012–2015, 2020, and 2022 at a depth of 10–13 m. The records with very small waves ($\sigma < 5$ cm) were not used in processing, which led to a significant reduction of the statistical ensemble. In total, we analyzed approximately 27,000 of 20-minute records, which corresponds to the time series of approximately 1 year.

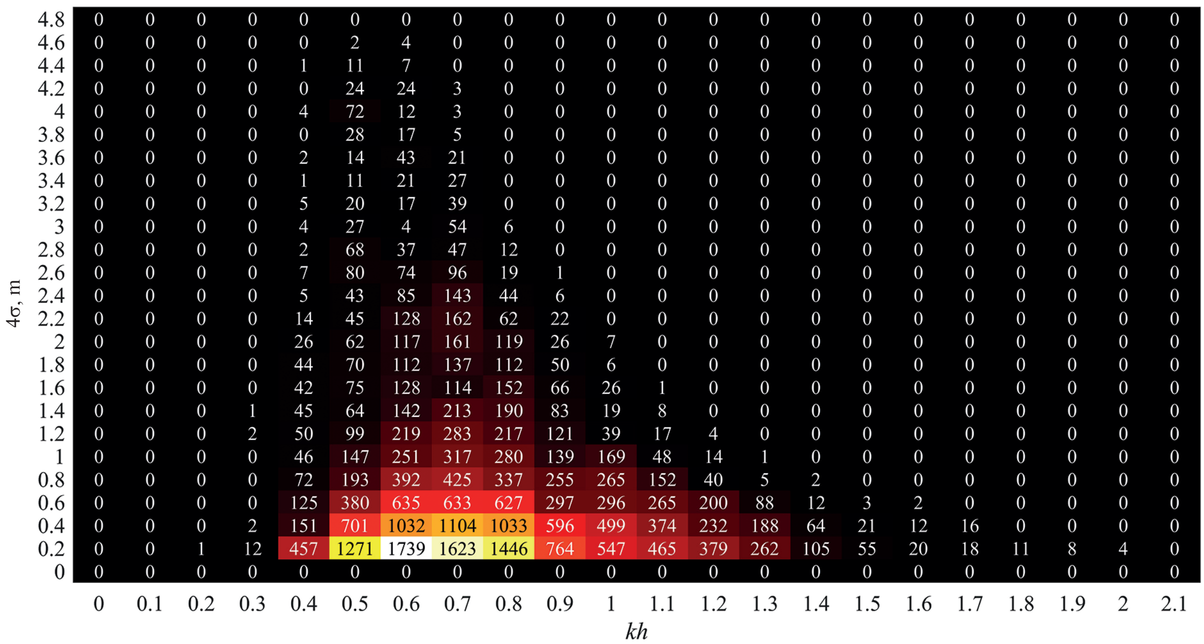
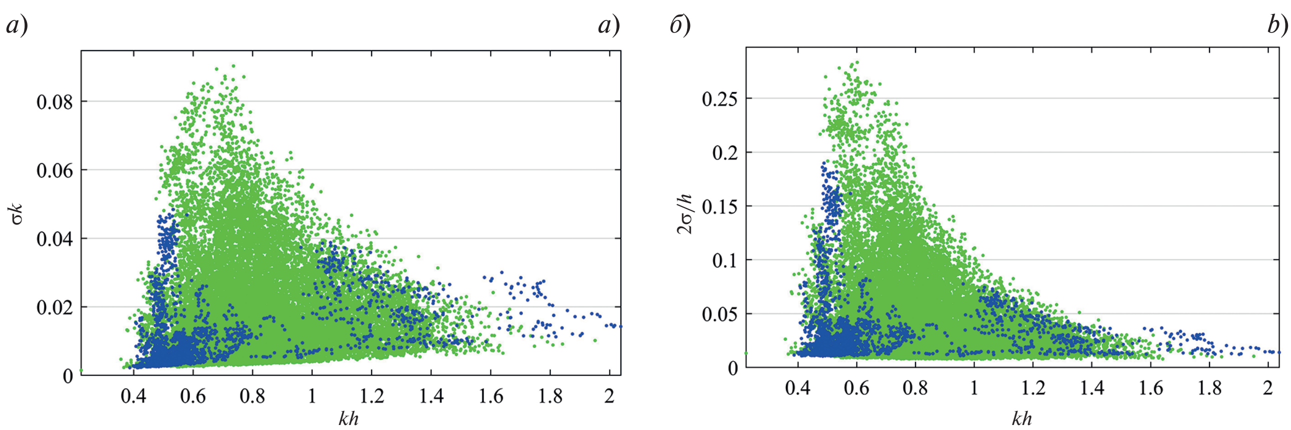


Fig. 3. Distribution diagram of 20-minute records number over significant heights H_s and dimensionless depths kh

Fig. 3 features a diagram similar to that shown in Fig. 2, but the values of the dimensionless depth kh are placed on the horizontal axis. As follows from the figure, variations in wave periods and local depth values lead to a large scatter of dimensionless depth values for different 20-minute records: from very shallow water $kh = 0.3$ to relatively deep water $kh = 2$.

The distributions of 20-minute records with respect to the depth parameter kh and one of the three nonlinearity parameters (4) are shown in Fig. 4. All distributions have a qualitatively similar form and demonstrate



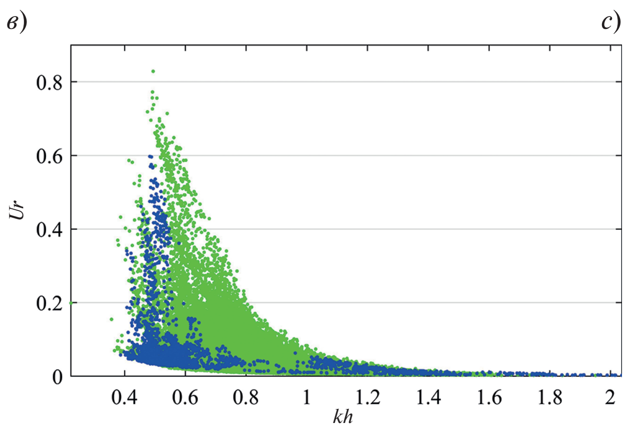


Fig. 4. Distribution diagrams of 20-minute records over dimensionless depths kh and nonlinearity parameters ε (*a*), *a* (*b*), and *Ur* (*c*). The dark blue dots are the periods of ice cover, the light green dots are the rest of the periods

events with a larger nonlinearity under the conditions of rather shallow water, although the position of the nonlinearity maximum is slightly different for the Figures 4a, 4b, and 4c: $kh \approx 0.7$, $kh \approx 0.6$, and $kh \approx 0.5$, respectively. Here, one should also consider the fact that most of the data correspond to this interval of dimensionless depths.

Fig. 4a indicates that the wave steepness is lower in ice conditions (dark blue dots) compared to the waves on the open surface (light green dots), although this may be the result of having significantly less data corresponding to ice conditions. On all panels in Fig. 4, the data in ice conditions are clustered in the interval of the smallest kh (long waves), while the nonlinear parameters a and Ur may not be small. A much smaller number of data (especially with significant nonlinearity) corresponding to the ice periods in the interval of average depths $kh = 0.6...1$ is striking, which has no physical interpretation yet.

An array of data sorted by dimensionless depth, as well as by one of the three nonlinearity parameters, was used to build probability distributions of wave height exceedances in several intervals of the corresponding parameter values. Thereby, we study the dependence of the probability distribution on one of four parameters: kh, ε, a, and Ur. These distributions are shown in Fig. 5; the corresponding intervals of values for each curve are also given there. The red dashed lines show the reference Rayleigh distributions.

The distribution of the number of 20-minute records by the considered parameter values is very unequal. When the general data array was divided into several subgroups containing approximately even number of waves, there were almost no differences for partial probability distributions. The number of waves corresponding to a significant deviation of the dimensionless parameter from the average value is small and provides a minor correction to the probability distribution. Therefore, the total data array was divided into subsets of different sizes that better reflect the different intervals of dimensionless parameters. The minimum probability value for the data in each subgroup is $1/N$, where N is the total number of waves. As can be seen from Fig. 5, the number of waves in the subsets under consideration may differ by an order of magnitude, but each of the subgroups has at least 10^5 of individual waves, which for the Rayleigh distribution would correspond to 30 rogue wave events.

The distributions in Fig. 5a show the probability distribution dependence on the dimensionless parameter kh. The distributions for $kh < 1.4$ are located closely. The sharp increase in the probability of the largest (with respect to $H_{1/3}$) waves for the interval of dimensionless depths $kh \le 0.7$ is probably associated with ice-time cases of abnormally high waves characterized by relatively long waves. Qualitatively, the distribution for $kh \le 0.7$ looks similar to Fig. 1a, where the heavy tail of extreme waves is associated with events of the ice period (see Fig. 1b and Fig. 1c). We would like to note the distribution for the deepest records for $kh \ge 1.4$, which is noticeably closer to the Rayleigh distribution than the other curves, including the range of not too rare events of $P < 0.01$.

The distributions in Figures 5b-5d demonstrate a qualitatively similar effect of a systematic decrease in the probability of waves with a great exceedance of the significant height with the nonlinearity parameter increase. For different intervals of relatively small wave steepness of $\varepsilon < 0.4$ (Fig. 5b), this behavior applies only to the very ‘tail’ of the distribution, but for $k\sigma \ge 0.04$, the probability of even relatively low waves decreases, $H > 1.2\ H_{1/3}$. According to the data in Fig. 5, the probability distribution changes most consistently and most sensitively when the shallow-water nonlinearity parameter a changes (Fig. 5c). In this case, the probability of events in the range $H \approx 2\ H_{1/3}$, depending on a, changes by an order of magnitude.

Fig. 5. Probability distributions of wave height exceedance by samples in the ranges of parameters: depth kh (a), wave steepness ε (b), shallow-water nonlinearity a (c), Ursell number Ur (d). The ranges of the corresponding parameters are indicated in the line codes

The distribution dependence on the parameter Ur, illustrated in Fig. 5d is not so clear, but the distributions for $Ur > 0.2$ and $Ur < 0.2$ are separated quite clearly. The results displayed in Fig. 5 indicate that field data show the strongest dependence of the probability distribution on a parameter with almost zero dependence on the dimensionless depth parameter kh (for a given data set providing a given range of values).

4. Conclusion

This paper analyzes the data of long-term waves measurements off the coast of Sakhalin Island in the Sea of Okhotsk by the bottom stations for recording pressure variations. These records are a part of a larger measurement database since 2009, which is planned to be investigated further. At this stage, the hydrostatic approximation was used to model the surface. Alternatively, consider the study applied to the original pressure records.

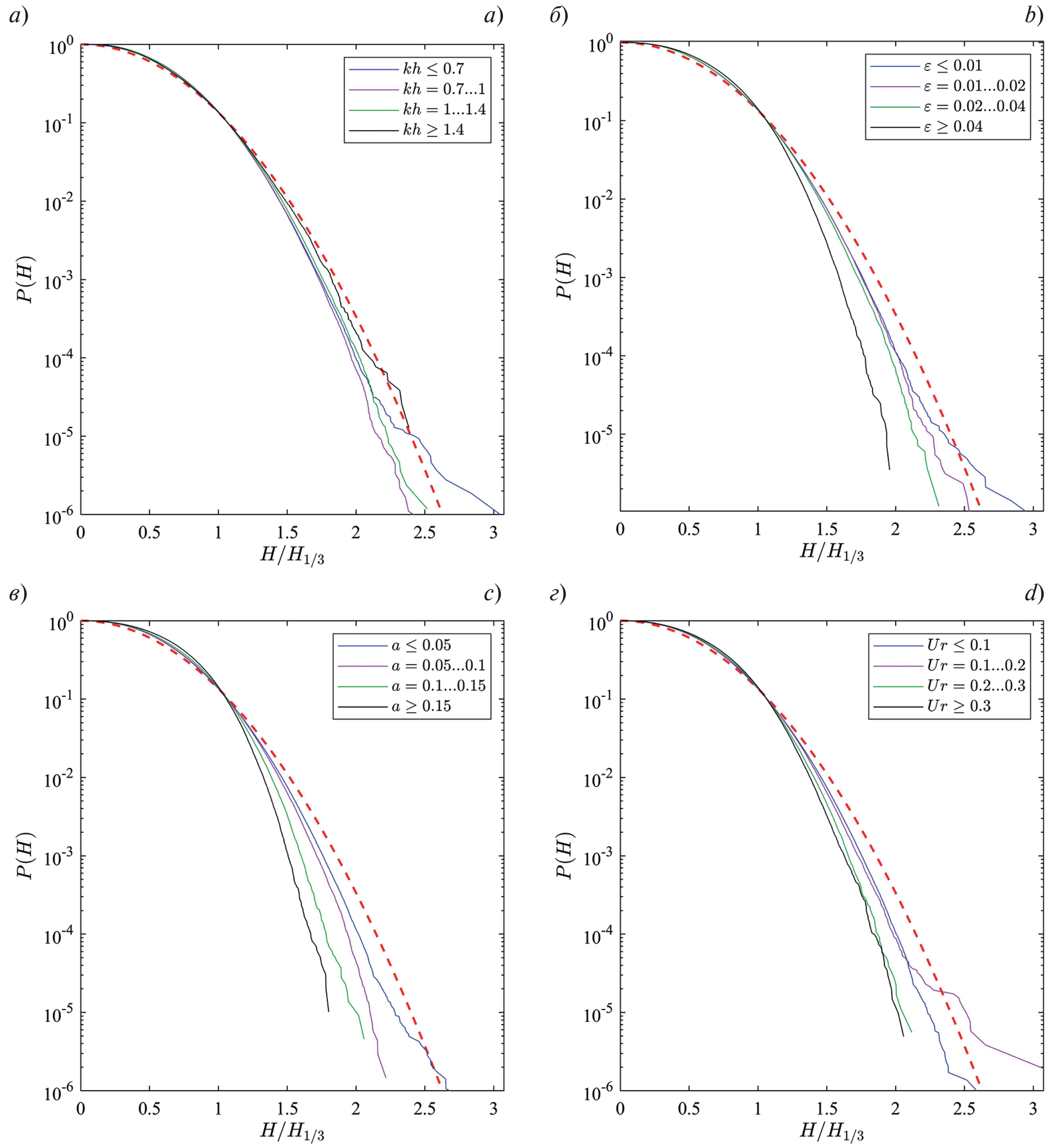


Fig. 5. Probability distributions of wave height exceedance by samples in the ranges of parameters: depth *kh* (*a*), wave steepness ε (*b*), shallow-water nonlinearity *a* (*c*), Ursell number *Ur* (*d*). The ranges of the corresponding parameters are indicated in the line codes

The main objective of the study was the division of the data into subgroups, which should correspond to physically equivalent conditions of wave propagation. This should help create representative subsets of statistically homogeneous waves and distinguish various physical processes involved in the formation of high waves. To sort the data, the natural dimensionless parameters of the task were used: the normalized depth of the measurement point and three nonlinearity parameters corresponding to the wave steepness (the nonlinearity parameter in deep water), the ratio of the wave amplitude to water depth (the nonlinearity parameter in shallow water), and the Ursell number (the ratio of the shallow water nonlinearity effects to dispersion). Despite the difference in physical meanings, all the listed nonlinear parameters are interconnected through the dimensionless depth.

Experimental probability distributions for subgroups of records corresponding to different intervals of dimensionless parameters were built. All dependences are to various extents different from the Rayleigh distribution: the experimental dependence lies above the theoretical curve in the range of small heights and below the Rayleigh distribution in the range of large wave heights. The events of the largest significant height exceedances observed during the ice period notably stand out from the general distribution and are characterized by a much higher probability. A significant dependence of the probability distribution function of wave heights on nonlinearity was revealed. It is visible most clearly when the shallow-water nonlinearity parameter (the ratio of the root-mean-square displacement to depth) changes. This conclusion is in qualitative agreement with the Glukhovsky theoretical distribution, for which the difference from the Rayleigh distribution is controlled by the same parameter, and the difference from the Rayleigh distribution looks qualitatively similar to the observations. With an increase in nonlinearity, the probability of waves exceeding the significant height by 2 times or more decreases.

Although most of the records correspond to the value $kh \approx 0.6...0.7$, due to the variation in the periods of incoming waves and tidal effects, according to the measurements at a fixed depth, it is possible to study waves in a wide range of dimensionless depths $kh = 0.3...2$. For them we can observe the dependence of the probability distribution on depth. For a sample of $kh \geqslant 1.4$, field data follow the Rayleigh distribution much better, showing an increase in the probability of high waves compared to the data for $kh < 1.4$.

Records that conditionally correspond to ice periods and contain waves with the greatest significant height exceedance are located on the planes of dimensionless parameters in the way different from other data. The waves in such records have a noticeable steepness of $k\sigma > 0.03$ in a fairly wide range of dimensionless depths ($kh = 0.5...1.2$). Significant nonlinearity in terms of the ratio of the root-mean-square surface displacement to depth and the Ursell parameter is observed only in a part of them, which contains relatively long waves.

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