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О ПРИРОДЕ ЭКСТРЕМАЛЬНЫХ ВОЛН В ОКЕАНЕ

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Одномерная конформная и трёхмерная фазоразрешающие численные модели использованы для исследования природы экстремальных волн в океане. Расчёты с конформной моделью проведены на период равный 7120 периодов волны пика. Небольшие потери энергии, возникающие за счёт её перехода в подсеточную область, компенсируется интегральным притоком энергии, так что полная энергия сохраняется с точностью до 4-х знаков. Рассчитаны вероятности полной высоты волны от подошвы до пика, а также возвышения от среднего уровня. Оценивается неопределённость (дисперсия) вероятности. Подтверждено, что вероятность полной высоты волны равной двум высотам характерной волны приблизительно соответствует высоте от среднего уровне равной 1.2. Экстремальные волны появляются случайным образом в виде групп, разделенных большими интервалами времени. Гипотеза, предполагающая, что экстремальные волны возникают как суперпозиция пиков нескольких мод в окрестности преобладающей волны, оказывается неверной. Специальным анализом распределения фаз доказано, что высота волны не коррелирует с плотностью концентрации фаз (представленной как сумма высот мод в окрестности пика доминантной волны). Вероятность высот больших волн монотонна по высоте. Это позволяет предположить, что возникновение больших волн является естественным свойством нелинейного волнового поля и такие волны представляют собой типичное, хотя и сравнительно редкое явление. Спектр волнового поля, состоящего из небольшого числа мод, может отражать присутствие экстремальной волны, но этот эффект полностью исчезает, когда длина такой волны значительно меньше, чем размеры области. Для Фурье аппроксимации экстремальной волны требуется много мод, как для аппроксимации импульсной функции. Быстрый рост экстремальной волны выглядит как фокусировка энергии в волне, сопровождающаяся концентрацией энергии в окрестности волнового пика. Можно предположить, что экстремальные волны близки по природе к бризерам.

Ключевые слова: экстремальные волны, неустойчивость Бенджамина–Фейера, суперпозиция волн, фазовый анализ, бризеры, Фурье-образ экстремальных волн.

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ABOUT NATURE OF EXTREME OCEAN WAVES

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The one-dimensional conformal model and the three-dimensional model for phase-resolving numerical simulation of sea waves were used for investigation of a freak wave nature. The calculations with conformal model were done for 7120 peak wave periods. The small subgrid dissipation of energy was compensated by integral input of energy, so, the energy was preserved with the accuracy of 4 decimal digits. The probability of the trough-to-crest wave height, crest wave height and trough depth were calculated. The uncertainty (dispersion) of the calculated probability is demonstrated. It is confirmed that trough-to-crest height equal to 2, approximately corresponds to the crest height equal to 1.2. Freak waves appear randomly in a form of the groups separated with large intervals of time. The hypothesis that freak wave can appear as a superposition of modes gathering in the vicinity of a dominant wave crest turned out to be incorrect. It was proved by a special phase analysis with the conformal model and 2D-model that the height of wave does not correlate with the phase concentration (expressed as a sum of crest heights of modes in the vicinity of crest of the main mode). The probability of large waves is monotonic over the wave height. It allows us to suggest that large waves are an indigenous property of a random wave field, and they are typical though quite rare events. The spectral image of wave field with a small number of modes can indicate the presence of freak waves, but this effect disappears completely when the length of such wave is much smaller than the size of domain. The Fourier approximation of freak wave in such domain requires many

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spectral modes in the same way as the approximation of pulse function. The abnormal growth of wave height looks rather like the self-focusing of single wave involving concentration of energy in the vicinity of wave peak. The breathers most closely correspond to the nature of freak waves.

Key words: Freak waves, Benjamin-Feir instability, superposition of waves, phase analysis, breathers, Fourier image of freak waves.

1. Introduction

In current paper the freak waves in open deep ocean with no currents are considered. The most popular theory for explanation of a freak wave phenomenon is the so-called '*modulational instability theory*' [1] originally known as '*Benjamin-Feir (B.-F.) instability theory*'. The concept of this theory is quite transparent, i. e., the one-dimensional nonlinear wave in presence of certain disturbances can produce additional modes arising in the vicinity of a main mode. Roughly speaking, the B.-F. theory explains redistribution of wave energy in frequency space up to the final homogenization of the initially discrete spectrum. Many scientists believe that this mechanism can explain abnormal growth of selected modes. In case of a broad spectrum typical for the wind-generated waves, such explanation is difficult to accept. First, it is unclear why one mode enjoys such preference and why this mode preserves its individuality in the course of its long development in a wave field with random phases. The original B.-F. results, as well as the numerical investigations of B.-F. [2] showed that the period of new mode growth for the typical sea wave steepness exceeds hundreds or thousands of the carrying wave period. Thus, freak wave should undergo a long course of development. Why do not the interactions with other waves stop this growth? Note that in many thousands of our numerical experiments with 1D and 2D-models such process of growth of a single mode or a dense group of modes was never registered. The authors of B.-F. instability call this process 'disintegration' probably keeping in mind that it does not have any creative functions.

The modulation instability theory of freak waves deals with such a vague characteristics as the Benjamin-Feir Index (BFI) [3], the parameter calculated as a ratio of wave steepness AK_p (A is wave amplitude at spectral peak and K_p is its wave number, both being dimensional), to the spectral bandwidth $\Delta K/K_p$, ΔK being a measure of width of the spectrum estimated as the half-width at the half-maximum of spectrum. Actually, the amplitude A at spectral peak essentially depends on spectral resolution. The value of 'width' of spectrum is also uncertain since wave spectrum normally embraces a wide range of frequencies, so the value of BFI finally depends on somewhat arbitrary quantities.

Anyway, it would be worth to emphasize that the modulation instability theory is an essentially spectral theory. The spectral presentation seems to be effective when it describes a more or less uniform process like a nonlinear interaction of waves (actually severely simplified quadruplet interactions) or energy input to waves, while it is rather pointless when applied to the analysis of extremely rare events represented by the single or isolated multi-peak disturbances of a vast wave field. Such disturbances are evidently created locally in a physical space while they cannot manifest themselves in a wave spectrum that characterizes a large area.

No detailed data on time/space development of large waves are available, however, the results of the 2D and 3D-mathematical modeling based on full equations show that the process of 'freaking' is very fast while the period of life of extreme waves is short. Such data do not prove an importance of the modulational instability theory for explanation of a freak wave phenomenon. This problem was discussed and illustrated by the numerous numerical results in [4, 2, 5]. They concluded that freak wave develops too fast to be explained by B.-F. theory (see also [6, 7].

The suggestion that freak waves can appear as a result of superposition of different modes seems more realistic. Such theory can explain why freak waves are rare and why their life time is short. Besides, the merging of crests can be followed by the focusing of energy. The superposition was used for reproducing wave breaking in wave channels, but role of this effect in generation of freak waves was not investigated systematically.

Probably, the approach most close to the nature of freak waves suggests solution of Nonlinear Schrödinger (NLS) equation [8–13] known as breathers emerging as isolated large-amplitude disturbances in a weakly nonlinear field with narrow spectrum. Such process was observed in calculations of waves with 1D-version of HOS model [14]. Such disturbances can fluctuate in time and space but they preserve their individuality long enough. This solution was indicated as a principal explanation of ocean freak waves [15–16]. The NLS equation is a simplified version of the initial Euler equations for potential flow with free surface. It is shown in this paper that a similar phenomenon can predict full nonlinear 2D-equations in the conformal coordinates (see also [17] and entire Volume 5 of Fundamentalnaya i Prikladnaya Gidrofizika, 2012).

2. Freak wave definition and simulations

Freak wave is defined as a wave whose trough-to-crest height exceeds twice the significant wave height H_s ($H_s =$ $= 4\sigma$ is the significant wave height, σ is the dispersion of surface). Since wave field actually consists of many 'modes' and looks chaotic, the definition assumes a use of algorithm of determination of the closest minimum and maximum points of elevation, and considers the difference between them as the trough-to-crest wave height. Unexpectedly it was found that the statistics of the trough-to-crest height of linear and nonlinear waves at the same spectrum is exactly identical [4, 18]. It means that the nonlinearity of wave field appears in vertical asymmetry of waves: the troughs are smoother and the crests are sharper than those for harmonic waves. The numerical investigation of freak waves by means of direct simulation can be done with a use of two-dimensional models like HOS model [19] or the models based on direct solution of the equations for velocity potential [20]; as well as with a one-dimensional model based on conformal mapping [21]. The last method is preferable, not only because this method is about thousand times faster than any method based on 2D-model, but mainly because it gives more precise statistics of extreme waves. All the 2D-numerical models based on the straightening of surface impose restriction on the local steepness of surface and thus understate the height of large waves. The conformal model can reproduce the steepening of waves up to a vertical wall, which allows simulating growth and sharpening of wave up to the breaking.

The aim of this work is investigation of spectral image of freak waves. According to the modulation instability theory, freak wave preserves its identity both in physical and spectral presentations. This statement is easy to check by analysis of the results of a long-term simulation of wave field for reasonably steep waves.

For calculation a conformal model [21] for deep water was used. This model is described in many papers and book [22]. The initial amplitudes were assigned with JONSWAP spectrum [23] for the inverse wave age $U/c_p = 1$ (U is wind velocity, c_p is phase velocity of peak wave). The calculations were done with 500 Fourier modes up to the time corresponding to 7120 peak wave periods t_p. The wave number of spectral peak was equal to 20. A very slow attenuation of total energy was compensated by input energy, so, the energy during the entire period was preserved with high accuracy. The statistical properties of a simulated wave field are illustrated in fig. 1 where the integral probabilities P_+ and P_- of the nondimensional positive Z_c and negative Z_t inclination of surface as well as the probability of trough-to crest height $P(Z_{tc})$ are given.

Abscissa axes correspond to the wave heights normalized by the significant wave height. The probability of Z_{tc} was calculated with a use of moving window [2]. As seen, the trough depths are considerably smaller than the heights of wave crests. On the average, the height of waves above mean sea level $Z_c = 1.2$ corresponds to trough-to-crest height, so, the condition $\tilde{Z}_0 = Z_0 / H_s = 1.2$ can be used as a criterion for recognition of freak wave.

Note that the data on probability of wave height contain uncertainty because it is not always clear what event should be considered as the single freak wave. The straightforward way consists of calculation of the portion of all the records with freak waves in the total volume of the data. However, some records can belong to the single moving freak waves. The cause of this uncertainty is the absence of strict definition of freak wave being either a case or a process.

It was proved in [4] that probability of trough-to-crest heights $P(Z_{tc})$ for linear and nonlinear waves is the same. (The vertical asymmetry of waves qualitatively explains the similarity of statistics of trough-to-crest wave height for linear and nonlinear wave fields, but it cannot explain their close identity [18]. This is probably a non-trivial consequence of conservation of dispersion (potential energy). The current calculations prove that the probability of freak waves (i. e., a wave with trough-to-crest height $Z_{tc} \geq 2$) is not too small and is equal to 5×10^{-4}. Since the total size of ensemble was around 5×10^6, the number of trough-to crest freak waves was equal to 250. Note that the integral probability

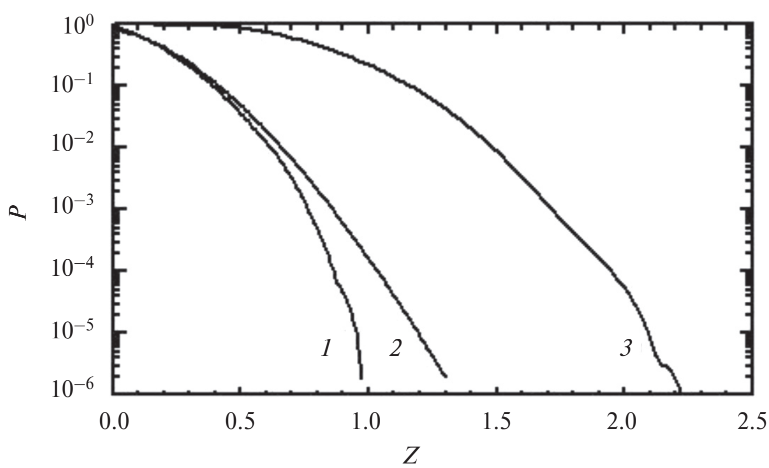

Fig. 1. Cumulative probability of trough depths P_- (*curve 1*); crests height P_+ (*curve 2*) and trough-to-crest height $P(Z_{tc})$ (*curve 3*).

Z_{tc} for two-dimensional waves is larger by 1–2 decimal orders than that for one-dimensional waves. This difference can be explained by increase of freedom while choosing a minimum/maximum pair in the rectangular window: the positions of the extremes in window do not coincide with the direction of wave propagation. When the probability of H_{ct} is calculated strictly along wave propagation, it turns out to be close to the probability for unidirected waves [4]. It follows that the probability of freak waves in 1D and 2D-wave fields is approximately the same.

It should be noted that the cumulative probability similar to that shown in fig. 1 is valid for quite large ensembles of data including space and time distribution. The probability can be also calculated for an individual wave field of size $N = 8000$. The results of such calculations are given in fig. 2, calculated by results of simulation [2], where the number of points falling in a cell is shown. As seen, individual fields can include up to 50 freak waves cases with $H_{tc} > 2$, or not include them at all.

The connection between H_{tc}, H_c and H_t is also valid in the statistical sense only: for example, wave peak with height $H_c = 1$ can belong to a wave with the total height $H_{tc} = 2.2$, because this specific wave has a deep trough. Oppositely, a wave with the crest height $H_c = 1.4$ does not necessarily belong to the trough-to-crest freak wave. It means that statistics of freak waves can be different for different criteria. It makes an impression that in some works the total height of freak waves H_{tc} was calculated simply by the doubling of crest height H_c, which is certainly incorrect.

The simulation with conformal model described above was used for separation of events of freak waves designated by the criteria $Z_{tc} > 2.00$. The sampling was done with interval $\Delta t = 0.01$, i. e. 14 times for one wave period. The events are shown in fig. 3 where horizontal axis corresponds to time T_p expressed in peak wave periods $T_p = 2\pi\left|k_p\right|^{-1/2}$, $k_p = 20$. Waves are shown as the vertical segments of length Z_{tc} with the bottom and top tips corresponding to Z_t and Z_c. The dots mark total height Z_{tc}. The segments show the location of freak waves in time, regardless of their location in space. As seen, the freak waves form compact groups. It is found that all the freak waves in the group are located closely to each other but they can temporarily attenuate when the value of Z_{tc} drops below 2. For the entire interval of calculations ($7120T_p$) eleven periods of high activity of freak waves were observed (table). The longest period (No 11) lasts $34T_p$.

The total duration of the groups containing freak waves is 85. The total number of freak waves registered with the interval $\Delta = 0.07T_{kp}$ is 334. Most of these events are attributed to a single freak wave. The average life time of freak waves is about one peak wave period T_{kp}.

As it is shown below, long groups contain different though somehow connected with each other extreme waves. The probability of freak waves demonstrated in fig. 1 should be understood as a ratio of the time during which such waves existed somewhere in the domain, to the total time of observation. Naturally, with increase of domain size the portion of freak waves will increase. (The probability of freak wave somewhere in the World Ocean is strictly equal to 1).

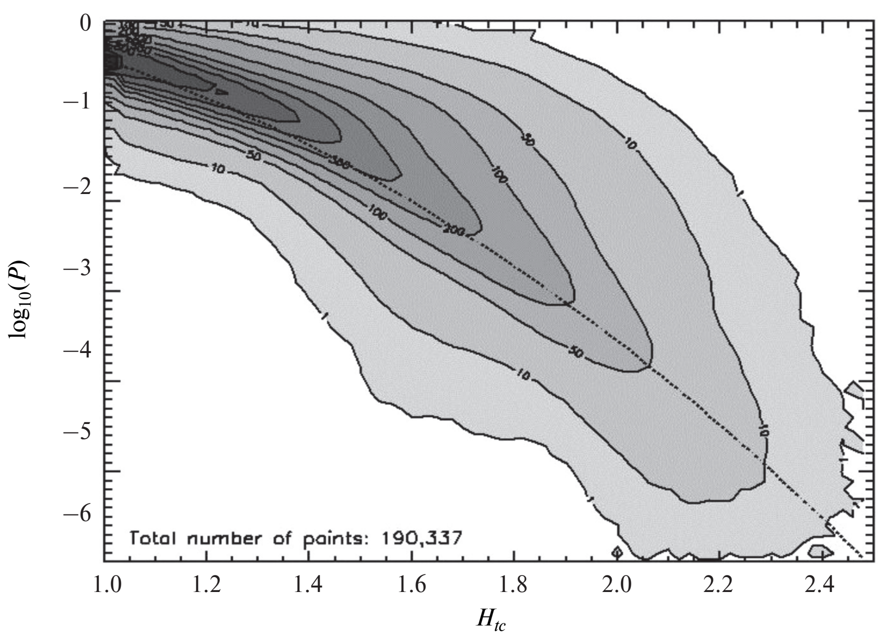

Fig. 2. Probability of H_{tc} calculated for 1D-wave profiles, each of them including 8000 points. Contours correspond to the number of cases falling into the cells with sizes $\Delta H_f = 0.02$ and $\Delta\log_{10}P = 0.01$ Dotted line is an averaged value for each bin ΔH_f.

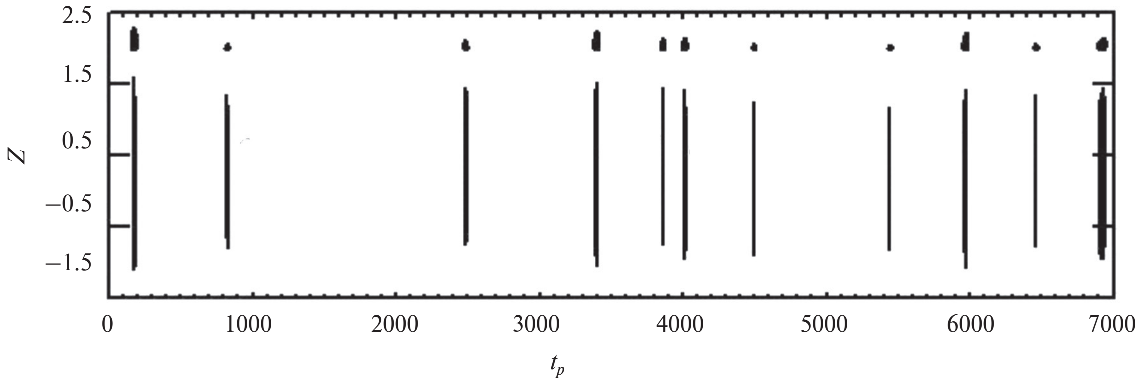

Fig. 3. The time table of freak waves ($Z_{tc} \geq 2.00$) shown as the vertical segments of length Z_{tc} with the bottom and top tips corresponding to Z_t and Z_c. Abscissa corresponds to time expressed in peak wave period. The dots in top of the panel indicate the value of Z_{tc}. The duration of each period is given in table.

Table

Characteristics of the groups shown in fig. 3: T_p^i is the time of appearance of wave with height $Z_{tc} \geq 2.00$; ΔT_p is a duration of the group (both are expressed in peak wave periods); Z_{tc}^m is the maximum trough-to-crest height in the group

No	1	2	3	4	5	6	7	8	9	10	11
T_p^i	171	825	2486	2491	3394	3861	4012	4498	5967	6463	6904
ΔT_p^m	11	4	2	1	14	4	10	2	11	2	34
Z_{tc}^m	2.25	2.02	2.07	2.03	2.18	2.09	2.11	2.01	2.18	2.02	2.10

It was suggested in [4] that freak wave can appear as a superposition of different modes. To check up this statement, the special processing of a great amount of wave profiles containing freak waves was performed. Firstly, the phases for every peak of each Fourier mode were calculated. Then all the amplitudes of modes falling in the same x-positions were summarized for the entire period of integration (7120 peak wave periods). Finally, the fields of 'phase density' P expressed in total sums of mode amplitudes for each cell of domain were calculated. If all crests of modes were incidentally concentrated in one point, then the crest height of wave would be equal to the sum of all amplitudes of modes (in our case it is approximately $1.7H_s$). Then the 'phase density' was averaged in the close vicinity of each peak of freak wave (denoted by $\overline{P}_f$) and normalized by the averaged over all domain phase density $\overline{P}$: $\overline{P}_n = \overline{P}_f / \overline{P}$.

The value of $\overline{P}_n$ describes the relative excess of the local phase concentration above the averaged over the phase density of the entire domain. The dependence of Z_{tc} on $\overline{P}_n$ is shown in fig. 4. Most of the points fall on the small values of $\overline{P}_n \sim 1$. Large values of $\overline{P}_n$ are distributed irregularly, and Z_{tc} does not show any tendency for increase with growth of $\overline{P}_n$. This result does not depend on the choice of width defining 'vicinity' of wave peak.

The calculations described above were repeated for a set of two-dimensional wave fields. For analysis 1100 wave fields were used with resolution 1024 × 2048 points generated by the model described by Chalikov (2016). The number of modes was 256 × 512, the peak wave number was equal to 20. The difference between the 1D and 2D-cases is that in a 2D-case the modes can come to any point under different angles. The phase density for peaks of all the modes was calculated in the 2D-vicinity of each peak found in the 'jumping window' with size 37 × 37 points. The algorithm allowed us to recognize nearly all of the local wave peaks. Such cumbersome calculations gave the same results as for one-dimensional waves: height of peaks turned out to be independent of phase concentration.

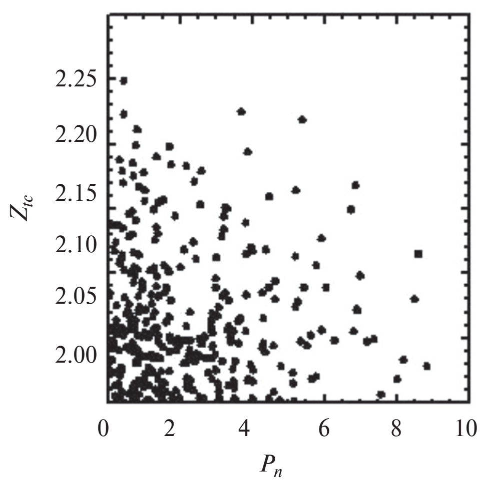

Fig. 4. Dependence of trough-to-crest wave height Z_{tc} on the relative 'phase density' $\overline{P}_n$.

The periods of high activity of freak waves as a rule contain a group of freak waves. It is well seen in the example of time/space distribution of elevation in fig. 5 (see Inset) where elevation is indicated with different colors (see legend and fig. 6 where the fragment of fig. 5 is given). Freak waves are marked with red dots which form short segments corresponding to an individual wave.

It is interesting to compare the evolution of wave field in fig. 5 with the evolution of spectrum in time/wave number space given in fig. 7 see Inset. The spectrum peak falls at wave number $k = 18$. During the first period of high activity of freak waves ($5980 < T_p < 5995$) the spectrum is closer to a single-peak shape. Starting from $T_p \approx 6030$ (second period) the spectrum obtains the three-peak structure because of the growth of two modes at wave numbers $k = 15$ and $k = 19$.

Time interval starting from $T_p \approx 6030$ is characterized by low freak wave activity (see fig. 5). The 1D-wave spectrum obtained by averaging over different periods is given in fig. 7.

The first period corresponds to high activity of freak waves while the second period contains no freak waves. As seen in fig. 8, additional modes at $k = 19$ and $k = 23$ appear in the second period. The amplitudes of these modes are even a little larger than the amplitude of carrying mode. Almost certainly these modes were developed as a result of Benjamin-Feir instability. After appearance of these modes the energy became distributed in k-space more uniformly and the conditions for freak wave formation became unfavorable. Hence, in this case the modulation instability rather reduces the number of freak waves since freak waves more likely appear at unimodal wave spectrum. This rule can be confirmed by the analysis of other groups shown in fig. 3.

It is reasonable to suggest that freak wave appears close to the peak wave frequency, later being somehow enforced. It was discussed before that the mechanism of mode superposition should be rejected as not existing. The theory of modulation instability claims that one or several modes in the vicinity of carrying mode, begin developing abnormally, taking the energy from the spectral environment. The problem is that this effect is difficult to investigate in a spectral space. Even if such mechanism does exist, it can be observed when the total number of modes is small and the physical domain is limited, which is typical for laboratory conditions. In real ocean the spectrum characterizes large space, and any single event of extreme wave cannot be pronounced in Fourier space.

The spectral composition of all 334 registered freak waves was investigated with special calculations. Firstly, amplitudes a_k and phases ϑ_κ for all 500 modes were calculated for every case. Then the elevations $\Delta z_{ik}(x_i)$ contributed by each Fourier modes were calculated:

$$z_{ik}\left(x_i\right) = a_k \cos\left(kx_i + \vartheta_k\right), \tag{1}$$

where x_i is the x-coordinate of freak wave peak with $Z_{tc} \geq 2.00$.

The nondimensional cumulative elevation Z_K normalized by the crest wave height Z_c

$$Z_c\left(K\right) = Z_c^{-1} \sum_{k=1}^{K} \Delta z_{ik}, \tag{2}$$

provided by the modes with $k \ll K$ as a function of K is shown in fig. 9.

The modes with wave numbers $k \leq k_p$ contribute only 30 % to the wave crest height; the modes with wave numbers $k \leq 2k_p$ give 70 %, the modes with wave numbers $k \leq 4k_p$ give 90 %. Negative values of Z_c in a vicinity of $K = 10$ mean that low wave number modes are not in phase with phase of maximum in given point. Wave becomes a true freak wave when

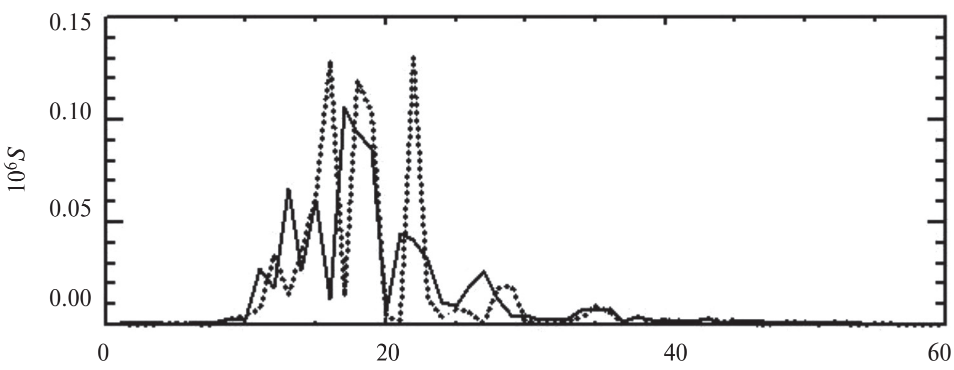

Fig. 8. Wave spectrum obtained by averaging over intervals ($5980 < T_p < 5993$) (*solid curve*) and ($6036 < T_p < 6042$) (*dotted curve*).

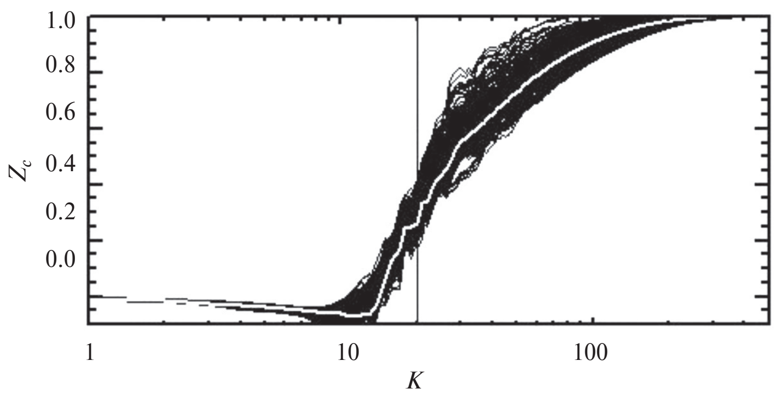

Fig. 9. Cumulative contribution Z_c of modes with wave numbers $k \le K$ normalized by crest height Z_c as a function of K. Vertical line shows peak wave number $k_p = 20$. White curve is the result of the averaging over all 334 events.

the remaining 10 % of the high-wave-number modes contribute to its height. Hence, we could come to a conclusion that in orchestration of freak wave the entire wave spectrum participates. It is nonsense, of course. Fourier series exactly approximates any surface, but isolated perturbations require many high wave number modes for their approximation.

3. Conclusion

Adiabatic equations of wave motion are self-similar, i. e. after their normalizing by use of the significant wave height and acceleration of gravity, for example, the statistical characteristics of waves become universal for the same nondimensional initial conditions. However, a pure adiabatic motion does not exist since the nonlinearity of equations produces transformation of spectrum and loss of energy due to the energy tendency to leave the computational domain. Since such processes are very slow, it is possible to support the total energy and consider the motion as adiabatic or quasi-adiabatic. Such method was used in the current paper for generation of a more or less uniform ensemble of wave surfaces. This approach is very convenient since the nondimensional equations assume reproduction of infinite of situations that differ from each other by a single multiplicative parameter.

In fact, the engineering practice does not require the nondimensional results. The probability of nondimensional freak waves is high: one wave out of two thousand waves turns out to be freak. A dimensional freak wave with height of 1 m is definitely dangerous and can be obviously called “a monstrous wave” by inhabitants of the Lilliputian land. Such wave can be freak wave in a nondimensional space. Since the significant wave height provides quite a robust scaling, the really dangerous waves can appear in a stormy sea only when the wave energy is high enough. A widely known Draupner wave with the height above mean level equal to 18.5 was registered when the significant wave height was 12 m. The nondimensional wave height Z_c was equal to 1.54. The approximation of probability for the crest wave height presented in [18].

$$P(Z_c) = \exp\left(-3.97 Z_c - 4.02 Z_c^2\right) \tag{3}$$

gives the probability of such wave $P(1.54) = 1.6 \times 10^{-7}$. It is interesting to note that the probability of trough-to- crest height estimated using the data from the above cited paper should be around 10^{-5}. The contradiction can be explained by the anomalous shallow trough of Draupner wave. i. e. Z_t = 7.1 m, though, according to the statistical data, it should be about 9.5 m. Probably such nonstandard ratio of the crest height to trough depth can be explained by the nonlinearity due to the small depth H = 70 m.

Naturally, the probability of nondimensional freak waves is much higher than that of real waves, as the height of real waves is proportional to significant wave height. The stronger the averaged waves the higher freak waves. Since the dimensional wave with specific height H_c^0 can be generated with various probability at different significant wave heights, the cumulative probability P of $H_c > H_c^0$ should be calculated by summation over all the values of the significant wave height in the interval $0 \le Z_c \le 1.85$ (the values above 1.5 were not registered.)

$$P\left(H_c > H_c^0\right) = \sum_{H_s} P\left(Z_c > Z_0\right) P_s\left(H_s > H_s^0\right) \Delta H_s, \tag{4}$$

where ΔH_s is the interval to which value H_s is referred [11].

Considering the practical application of the theory of rare waves, we can also come to the conclusion that a conventional ‘definition’ of dimensional freak waves is not required at all. For better use of research recommendations, it would be more efficient to define the categories of freak waves, as it has been done, for example, for tropical storms. A reasonable warning on appearance of such waves should sound as follows: ‘from 6 am today until 6 am tomorrow in a specific area of 100 × 100 km^2 a wave as high as 10 m (category three) will be one of 1000 ± 200 waves; a wave with height of 15 m (category five) will be one of 8000 ± 1000 waves,... etc.’ The warning can be also expressed it terms of expecting time. The data on the scatter of wave probability demonstrated in fig. 2 show that with no indication of confidence intervals the forecast of extreme waves is senseless. The methods of wave forecasting should be different for different purposes. For example, for navigation purposes the forecasts should take into account duration of a forecasting period, while for climate research targeted at construction projects the estimation should give extreme values for long periods. The environmental conditions in different areas of ocean are different, depending on wind, currents statistics and bathymetry; hence, the forecast should be based on the local observations. Such data can also be obtained with a spectral wave model. In this case a method of interpretation of spectral data in terms of the wave height probability should be suggested [18]. For limited domains the statistical data on waves can be generated with the phase-resolving finite-difference models developed at Technical University of Denmark [24].

The nature of freak waves still remains unclear. The modulational instability is unable to explain the suddenness and short life of freak waves. The multiple numerical simulation of Benjamin-Feir instability [2] demonstrated an excellent agreement with the theoretical results, but never predicted an abnormal growth of separate modes significantly exceeding the mean wave height. Extreme waves appeared suddenly after the B.F. instability had filled up empty intervals of spectrum. The approximation of wave field by superposition of harmonic modes gives true statistics for trough-to-crest height only and significantly underestimates the probability of wave crests. The approximation of wave field by superposition of Stokes modes gives essentially the same results due to small steepness of real waves. Chalikov and Babanin (2016) put their faith in mechanism of superposition of different modes leading to extreme wave formation. A thorough analysis performed in the process of work on this paper proved that the density of phase concentration does not correlate with the local wave height; hence, the effect of superposition cannot be responsible for freak waves.

The question arises: why are freak waves considered as an outstanding phenomenon? The simulations with non-dimensional equations show that such waves are not so infrequent. Natural freak waves are rare only because the stormy conditions are exceptional. The definition of freak wave as a wave whose height exceeds $2H_s$ looks unjustified.

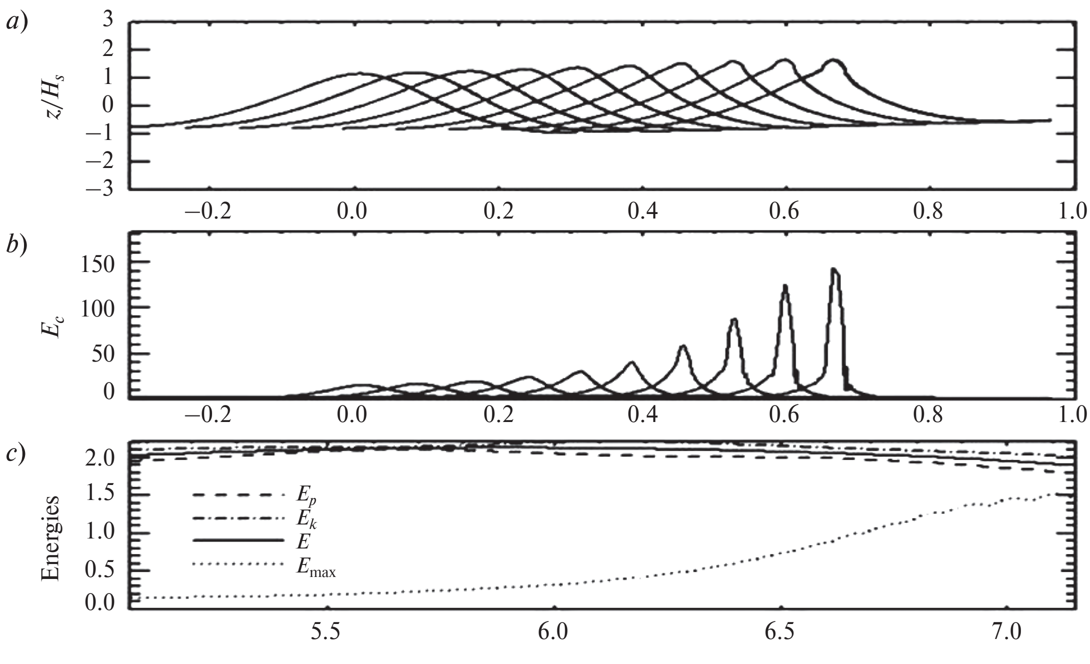

Fig. 10. Example of a simulated wave. In panels *a, b* the horizontal axis is a distance: panel *a* represents successive profiles (separated by the interval $\Delta t = 0.02$) of the largest wave within a time range from $t = 5.06$ ($H_f = 2.10$ at $t = 2.28$ periods) up to the overturning moment at $t = 7.15$ (3.22 periods); *b* — corresponding to (*a*) evolution of columnar energy e_c; panel *c* shows time evolution of maximum values of total E_m, columnar kinetic (E_k), potential (E_k) and total (E) energy.

Why only 2? On the average, there are 7 times more waves with height $1.9H_s$, while there are 7 times less waves with height $2.1H_s$. The probability of large waves is monotonic over wave height. It seems a lot more reasonable to suggest that large waves are an indigenous property of a random wave field, and they are typical though quite rare events. The spectral image of wave field with a small number of modes can indicate the presence of freak waves, but this effect disappears completely when the length of such wave is much smaller than the size of domain. Fourier approximation of freak wave in such domain requires many spectral modes in the same way as the approximation of pulse function.

The details of the extreme wave development are given in fig. 10 obtained in the process of simulations of freak waves with conformal model [21]. Freak wave is developing just over two wave periods. The energy in peak column grows approximately 10 times over this period of time. The evolution of energy averaged throughout the trough to trough interval (which is assumed to be overall energy of the chosen wave) as well as the maximum of energy at wave peak are given in panel *b*.

The most surprising feature of this picture is that the total energy of developing wave remains nearly constant (it cannot be an exact constant as the domain has open boundaries), while its peak value grows dramatically. In other cases, the total energy of certain waves is even slightly decreasing. It proves that the freak wave goes through a self-amplification phase with no substantial exchange of energy with other waves.

Therefore, any considerations of freak wave generation in Fourier space are pointless: just one wave in a wide set of similar waves unpredictably begins developing fast, accompanied by powerful concentration of energy in the vicinity of wave peak. Evidently, it is the main property of extreme wave, which makes the largest of them a freak one. The mechanisms of this evolution are still unknown; the prediction of time and location of wave development ("freaking") is impossible even in numerical experiments. Fortunately, enough, such knowledge would not make any sense for practical use.

Much more important are the statistics of such events and mechanical characteristics of freak waves. The above problem is similar to that of the numerical forecast of thunderstorms: the atmospherics model can predict a possibility of storm generation in cell of a numerical model, but not the exact location and time of such events.

In our opinion, the most likely cause of freak wave is the concentration of energy in a physical space. The Fourier transformation surely reflects this process just because it provides an exact approximation of surface, but interpretation of this image in terms of freak waves is possible only if the length of such wave is not too small as compared with the size of domain.

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К статье *Чаликов Д.В.* О природе экстремальных волн в океане
Chalikov D.V. About Nature of Extreme Ocean Waves

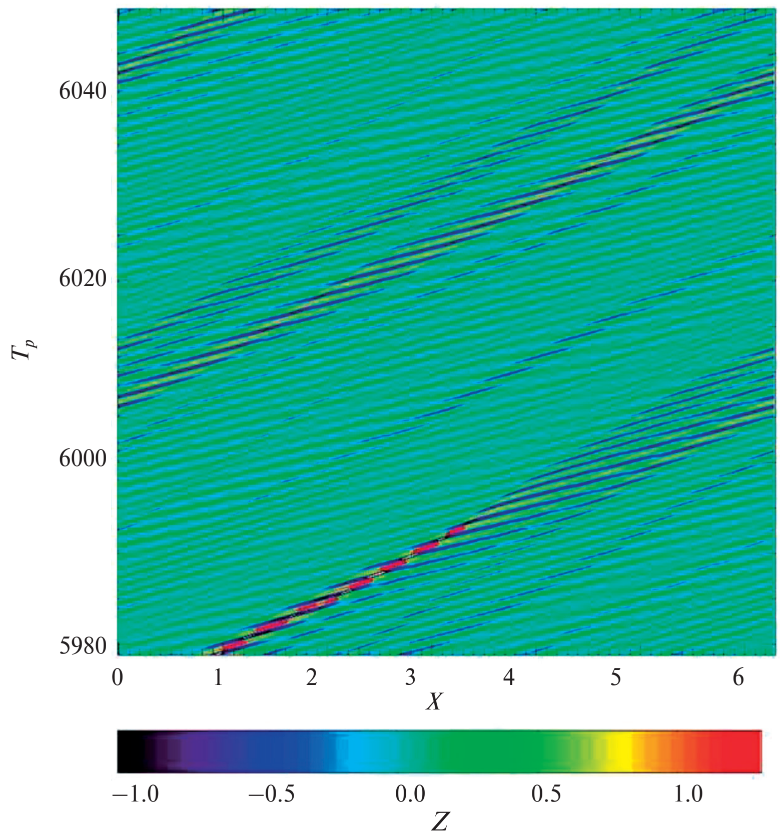

Fig. 5. Time and space evolution of elevation for the period $T = (5979–6050)\ T_{kp}$. Freak waves are marked with red color.

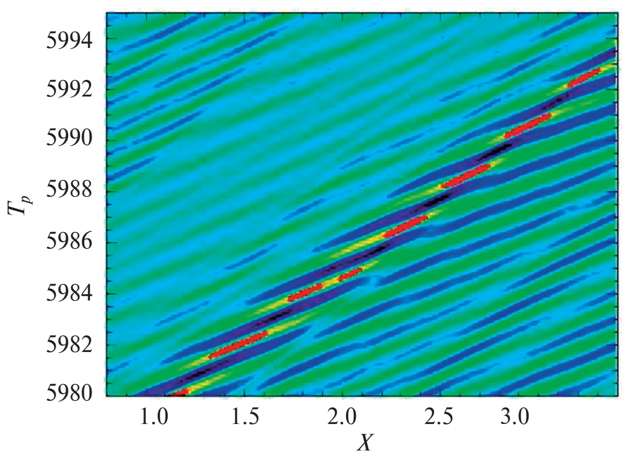

Fig. 6. Fragment of fig. 5 (its left bottom corner). It is seen that the entire group consists of individual freak waves remaining in 'freak capacity' for about one peak wave period. The entire group existed about 10 periods.

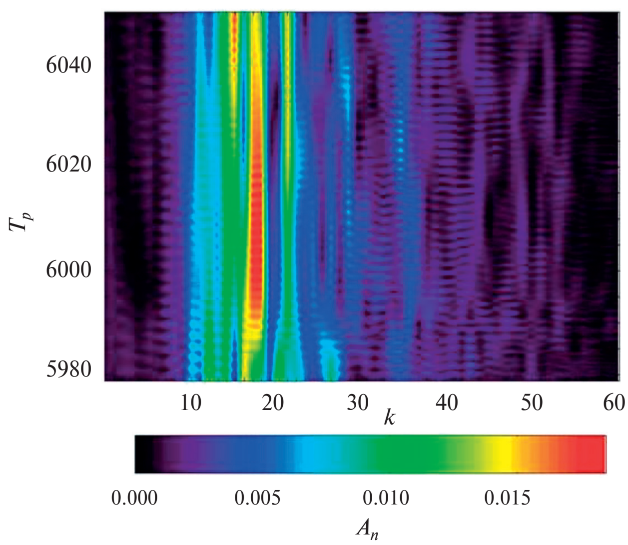

Fig. 7. Evolution of wave spectrum expressed in amplitudes of Fourier modes in time/wave number space during the period corresponding to that in fig. 5.